THE THEORETICAL PREDICTION
OF THE
CENTER OF PRESSURE

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CENTER OF PRESSURE CALCULATIONS

Background

The most important characteristic of a model rocket is its stability.

A rocket's static stability is affected by the relative positions of its center of gravity(C.G.) and its center of pressure(C.P.). As is well known, the static margin of a rocket is the distance between the C.G. and C.P. A rocket is statically stable if the C.P. is behind the C.G.; also, it is more stable for a larger static margin.

The center of gravity of a rocket is easily determined by a simple balance test. The center of pressure, determination is much more difficult. Many methods for determining the C.P. have been proposed. The majority of them boil down to the determination of the center of lateral area which is the C.P. of the rocket if it were flying sideways. The center of lateral area is a conservative estimate; that is, it is forward of the actual C.P.; and, as such, is not a bad method for the beginner. However, as model rocketry becomes more sophisticated, and rocketeers become more concerned with reducing the static margin to the bare minimum; to reduce weathercocking a more accurate method is called for.

The center of pressure is the furthest aft at zero angle of attack. By calculating the C.P. at $\alpha = 0$; therefore; one has the least conservative value. It is this value to which any safety margin should be added.

The advantage of this method is that it reduces the static margin to a safe and predetermined minimum.

The existance of an easily applicable set of equations for the calculation of the C.P. allows the rocketeer to truly design his

Background(cont.)

birds before any construction takes place. Since, by necessity, the derivation of any equations requires a predetermined configuration; a method of iteration must be used to determine the final design.

CENTER OF PRESSURE CALCULATIONS

Objective: To derive the subsonic theoretical center of pressure equations of a general rocket configuration. And to simplify the resulting equations without a great loss of accuracy so that the average leader can use them.

Method of Approach:

1. Divide rocket into separate portions.
2. Analyse each portion separatly.
3. Analyse the interference effects between the portions.
4. Simplify the calculations where necessary.
5. Recombine the results of the separate analyses to obtain the final answer.
6. Verify Analysis by experiment.

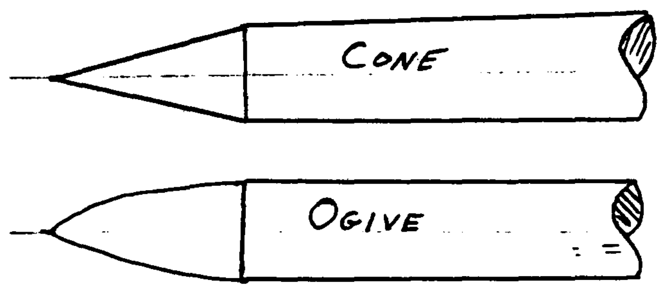
Assumptions:

1. Flow over rocket is potential flow. ie, no vortices or friction
2. Point of the nose is sharp.
3. Fins are thin flat plates with no cant.
4. The angle of attack is very near zero.
5. The flow is steady state and subsonic.
6. The rocket is a rigid body.
7. The rocket is axially symmetric.

Portioning of Rocket

A rocket is, in general, composed of the following portions:

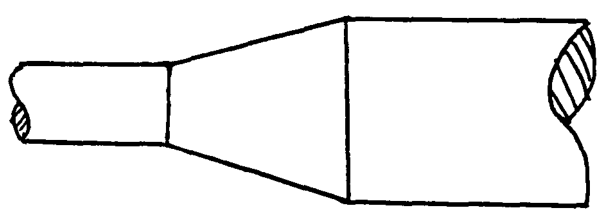
1. Nose



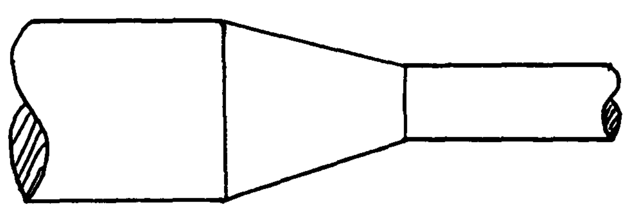
2. Cylindrical Body

Different diameters before and after any conical sholder..

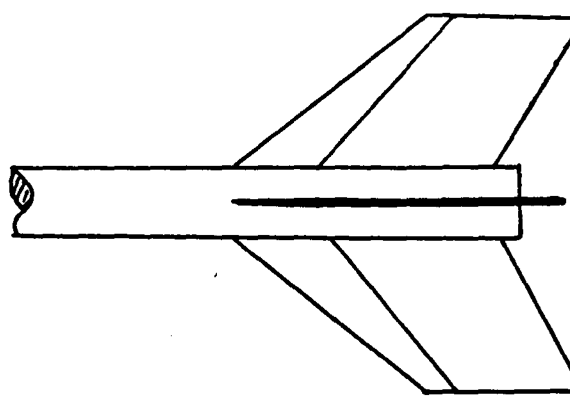
3. Conical sholder



4. Conical Boattail



5. Fins



Symbols

A = Reference Area = $\frac{\pi}{4} d^2$

A_f = Area of One fin

AR = Aspect Ratio

C = General Fin Chord Length

C_m = Nondimensional Aerodynamic Pitching Moment Coefficient = M/Aqd

C_{m_α} = Slope of Moment Coefficient Curve at $\alpha = 0$, $\partial C_m/\partial\alpha |_{\alpha=0}$

C_{MA} = Mean Aerodynamic Chord.

C_N = Nondimensional Aerodynamic Normal Force

C_{N_α} = Slope of the Force Coefficient at $\alpha = 0$, $\partial C_N/\partial\alpha |_{\alpha=0}$.

C_r = Root Chord Length

C_t = Tip Chord Length

d = Reference Length = Diameter at the base of the nose

F = Diederich's Correlation Parameter

f = Fineness Ratio

K = Interference Factor

L = Body Portion Length

ℓ = Length of Fin Midchord Line

$M(x)$ = Local Aerodynamic Pitching Moment About the Front of the Body Portion

$n(x)$ = Local Aerodynamic Normal Force

q = Dynamic Pressure = $\frac{1}{2}\rho V_0^2$

r_t = Radius of Body Between Fins

S = Fin Semispan

$S(x)$ = Local Crosssectional Area

V_o = Free Stream Velocity

$\mathcal{V}$ = Body Portion Volume

$W(x)$ = Local Downwash Velocity

x = General Distance Along Body

$\bar{x}$ = Center of Pressure Location

X_F = Distance Between the Nose Tip and the Leading Edge of the Fin Root Chord

X_t = Distance Between the Root Chord Leading Edge and the Tip Chord Leading Edge in a Direction Parallel to the Body

y = General Fin Spanwise Coordinate

$\bar{Y}$ = Spanwise Location of Mean Aerodynamic Chord

α = Angle of Attack

Λ_L = Sweep of Fin Leading Edge

σ = Sweep of Fin Midchord Line

λ = Fin Taper Ratio = C_t/C_r

ρ = Freestream Density

<u>Subscripts</u>

B = Body

F = Tail or Fins

N = Nose

CS = Conical Shoulder

CB = Conical Boattail

$T(B)$ = Tail in the Presence of the Body

BODY AERODYNAMICS DERIVATIONS

Normal Force Coefficient Slope

General:

For an axially symmetric body of revolution; from reference 4; the subsonic steady state aerodynamic running normal load is given by;

$$n(x) = \rho V_0 \frac{\partial}{\partial x}\left[S(x)\,w(x)\right] \qquad (1)$$

where; (See figure 1)

- $n(x)$ = The running normal load per unit length.
- ρ = Free stream density
- V_0 = Free stream air speed
- $S(x)$ = Local crossectional area
- $w(x)$ = Local downwash at a given point on the body.

A rigid body has the downwash given by;

$$w(x) = V_0 \alpha \qquad (2)$$

Thus;

$$n(x) = \rho V_0^2 \alpha \frac{\partial S(x)}{\partial x} \qquad (3)$$

By the definition of normal force coefficient;

$$C_N(x) = \frac{n(x)}{qA} = \frac{n(x)}{\frac{1}{2}\rho V_0^2 A} \qquad (4)$$

Substituting equation 3 into 4

$$C_N(x) = 2\,\frac{\alpha}{A}\,\frac{\partial S(x)}{\partial x} \qquad (5)$$

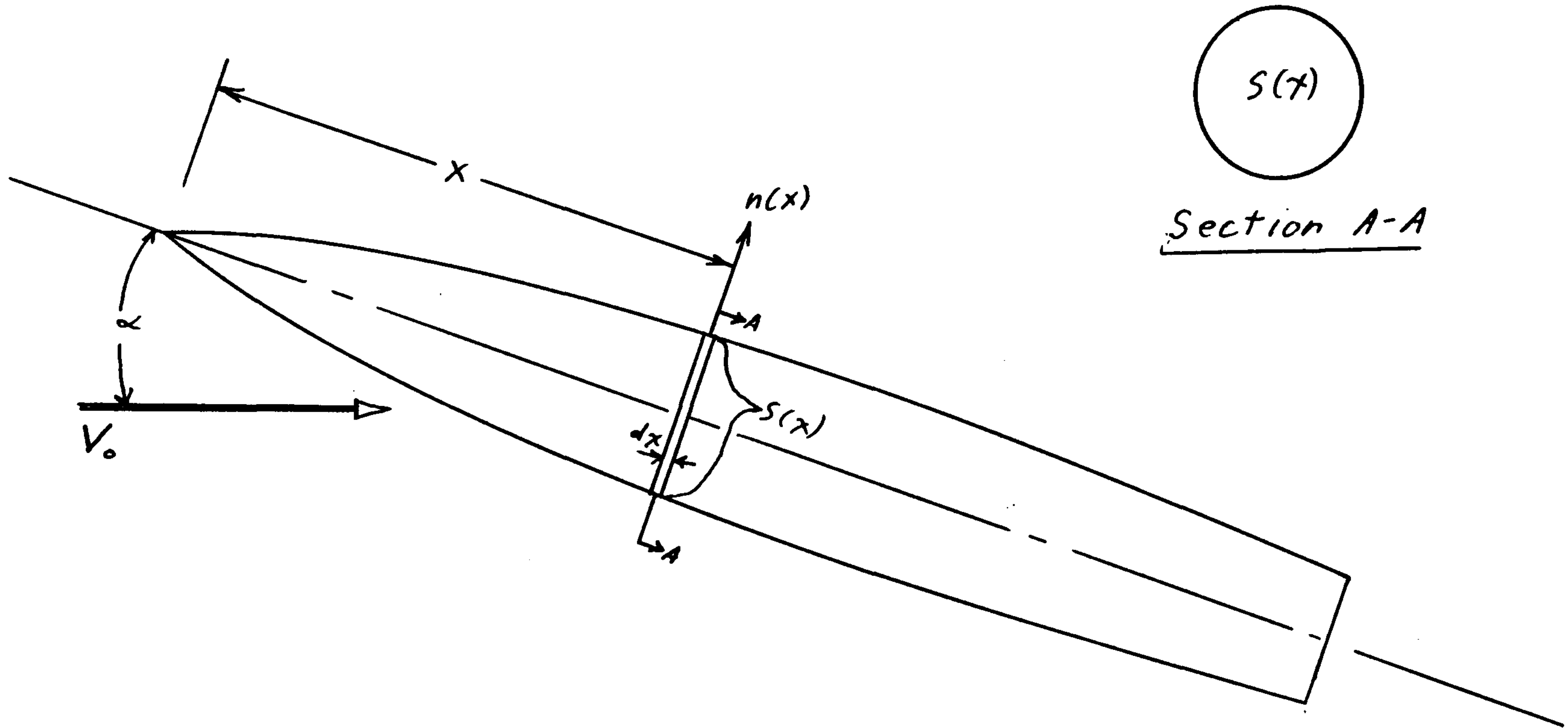


FIGURE 1 BODY STATION PARAMETERS

but;

$$A = \frac{\pi}{4} d^2$$

therefore;

$$C_N(x) = \frac{8\alpha}{\pi d^2} \frac{\partial S(x)}{\partial x} \quad (6)$$

By the definition of the normal force coefficient curve slope;

$$C_{N_\alpha}(x) = \left.\frac{\partial C_N}{\partial \alpha}\right|_{\alpha=0} = \frac{8}{\pi d^2} \frac{\partial S(x)}{\partial x} \quad (7)$$

In order to obtain the total C_{N_α}, Equation 7 is integrated over the length of the body;

$$C_{N_\alpha} = \int_0^L C_{N_\alpha}(x)\,dx = \int_0^L \frac{8}{\pi d^2} \frac{\partial S(x)}{\partial x}\,dx \quad (8)$$

Since $8/\pi d^2$ is not a function of x;

$$C_{N_\alpha} = \frac{8}{\pi d^2} \int_0^L \frac{\partial S(x)}{\partial x}\,dx \quad (9)$$

Performing the integration in 9; and noting that the antiderivative of;

$$\frac{\partial S(x)}{\partial x}$$

is;

$$S(x)$$

Then;

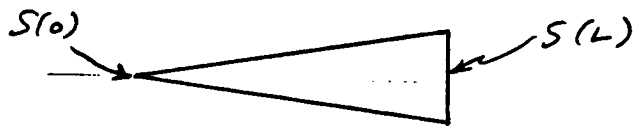
$$C_{N_\alpha} = \frac{8}{\pi d^2}\left[S(L) - S(o)\right] \qquad (10)$$

Immediatly it is noticed that C_{N_α} is independent of the shape of the body as long as the body is such that the integration is valid.

Equation 10 is now applied to the different portions of the body.

Nose

For the nose; $S(o) = 0$



Thus:

$$C_{N_\alpha} = \frac{8}{\pi d^2}\left[S(L) - 0\right] \qquad (11)$$

But;

$$S(L) = \frac{\pi d^2}{4}$$

Thus;

$$\boxed{(C_{N_\alpha})_N = 2} \text{ (per radian)} \qquad (12)$$

This result holds for ogives, cones, or parabolic shapes; as well as any other shape that varies smoothly.

Cylindrical Body

For any cylindrical body; $S(L) = S(o)$

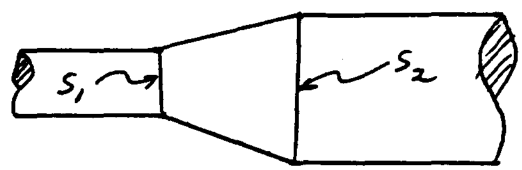
Thus;

$$C_{N_\alpha} = 0 \qquad (13)$$

This says that there is no lift on the cylindrical body portions at low angles of attack.

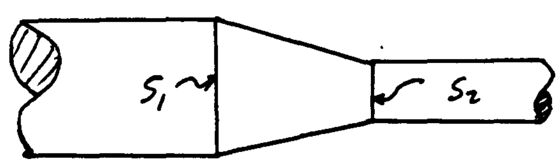
Conical Sholder

Equation 10 is directly applicable to both Conical sholders and boattails



$$(C_{N_\alpha})_{CS} = \frac{8}{\pi d^2}(S_2 - S_1) \qquad (14)$$

Conical Boattail



$$(C_{N_\alpha})_{CB} = \frac{8}{\pi d^2}(S_2 - S_1) \qquad (15)$$

Since S_2 is less than S_1, for a conical boattail, the value of $(C_{N_\alpha})_{CB}$ is negative for angles of attack near zero.

BODY AERODYNAMICS DERIVATIONS

Center of Pressure

General;

By definition, the pitching moment of the local normal aerodynamic force about the front of the body $(x=0)$ is;

$$M(x) = X\,n(x) \qquad (16)$$

Substituting equation 3 into equation 16 =

$$M(x) = \rho V_0^2 \alpha X \frac{\partial S(X)}{\partial X} \qquad (17)$$

By definition of the aerodynamic pitching moment coefficient,

$$C_m(x) = \frac{M(x)}{qAd} = \frac{M(x)}{\frac{1}{2}\rho V_0^2 A} \qquad (18)$$

Substituting equation 17 into equation 18;

$$C_m(x) = \frac{2\alpha X}{Ad} \frac{\partial S(x)}{\partial X} \qquad (19)$$

but;

$$A = \frac{\pi}{4} d^2$$

Therefore,

$$C_m(x) = \frac{8\alpha X}{\pi d^3} \frac{\partial S(x)}{\partial x} \qquad (20)$$

By the definition of moment coefficient curve slope;

$$C_{m_\alpha}(x) = \left.\frac{\partial C_m(x)}{\partial \alpha}\right|_{\alpha=0} = \frac{8x}{\pi d^3}\frac{\partial S(x)}{\partial x} \quad (21)$$

In order to obtain the total C_{m_α} equation 21 is integrated over the length of the body;

$$C_{m_\alpha} = \int_0^L \frac{8x}{\pi d^3}\frac{\partial S(x)}{\partial x}dx \quad (22)$$

Since $8/\pi d^3$ is not a function of x ;

$$C_{m_\alpha} = \frac{8}{\pi d^3}\int_0^L x\frac{\partial S(x)}{\partial x}dx \quad (23)$$

Performing the integration in 23 by parts;

$$u = x \qquad dv = \frac{\partial S(x)}{\partial x}dx$$

$$du = dx \qquad v = S(x)$$

$$C_{m_\alpha} = \frac{8}{\pi d^3}\left\{\Big[xS(x)\Big]_0^L - \int_0^L S(x)dx\right\}$$

$$= \frac{8}{\pi d^3}\left\{\Big[L\,S(L) - 0\,S(0)\Big] - \int_0^L S(x)dx\right\}$$

$$C_{m_\alpha} = \frac{8}{\pi d^3}\left[L\,S(L) - \int_0^L S(x)dx\right] \quad (24)$$

By definition, the second term in 24 is the volume of the body;

$$\mathcal{V} = \int_0^L S(x)\,dx \qquad (25)$$

Thus;

$$C_{m_\alpha} = \frac{8}{\pi d^3}\left[L\,S(L) - \mathcal{V}\right] \qquad (26)$$

The center of pressure of the body is defined as;

$$\bar{X} = d\left(\frac{C_{m_\alpha}}{C_{N_\alpha}}\right) \qquad (27)$$

Substituting equations 10 and 26 into equation 27;

$$\bar{X} = \frac{L\,S(L) - \mathcal{V}}{S(L) - S(0)} \qquad (28)$$

Dividing numerator and denominator by $S(L)$;

$$\bar{X} = \frac{L - \mathcal{V}/S(L)}{1 - \frac{S(0)}{S(L)}} \qquad (29)$$

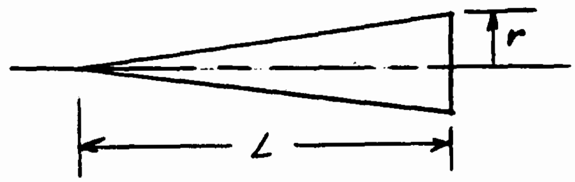
The center of pressure, then, is a definite function of the body shape which determines the volume.

Equation 29 is now applied to the different portions of the body.

Nose

The nose shapes most often used are that of either a cone or an ogive. Thus; $\bar{X}$ is determined for those particular shapes.

Cone



$$v = \frac{\pi}{3} r^2 L = \frac{1}{3} L\, S(L)$$

Thus;

$$v/S(L) = L/3 \qquad (30)$$

also; $S(0) = 0$
thus;

$$\frac{S(0)}{S(L)} = 0 \qquad (31)$$

Therefore;

$$\bar{X} = \frac{L - L/3}{1 - 0}$$

or,

$$\boxed{\bar{X}_N = \frac{2}{3} L} \quad \text{CONE} \qquad (32)$$

Ogive

From reference 2; for a tangent ogive,

$$\frac{v}{4d\,S(L)} = f\left(f^2+\tfrac{1}{4}\right)^2 - \tfrac{1}{3} f^3 - \left(f^2-\tfrac{1}{4}\right)\left(f^2+\tfrac{1}{4}\right)^2 \sin^{-1}\left(\frac{f}{f^2+\frac{1}{4}}\right) \qquad (33)$$

where;

$$f = \frac{L}{d} \qquad (34)$$

Again; the denominator is 1, since $S(0) = 0$.
Thus;

$$\bar{X} = L - \frac{v}{S(L)} \qquad (35)$$

Dividing equation 35 by d ;

$$\frac{\bar{X}}{d} = f - \frac{v}{d\,S(L)} \qquad (36)$$

or, substituting equation 33 in equation 36

$$\frac{\bar{X}}{d} = f + 4\left[f(f^2+\tfrac{1}{4})^2 + \tfrac{1}{3}f^3 + (f^2-\tfrac{1}{4})(f^2+\tfrac{1}{4})^2 \sin^{-1}\left(\frac{f}{f^2+\frac{1}{4}}\right)\right] \qquad (37)$$

$$f + 4\left[-f(f^2+\tfrac{1}{4})^2 - \tfrac{1}{3}f^3 - (f^2-\tfrac{1}{4})(f^2+\tfrac{1}{4})^2 \sin^{-1}\left(\frac{f}{f^2+\frac{1}{4}}\right)\right]$$

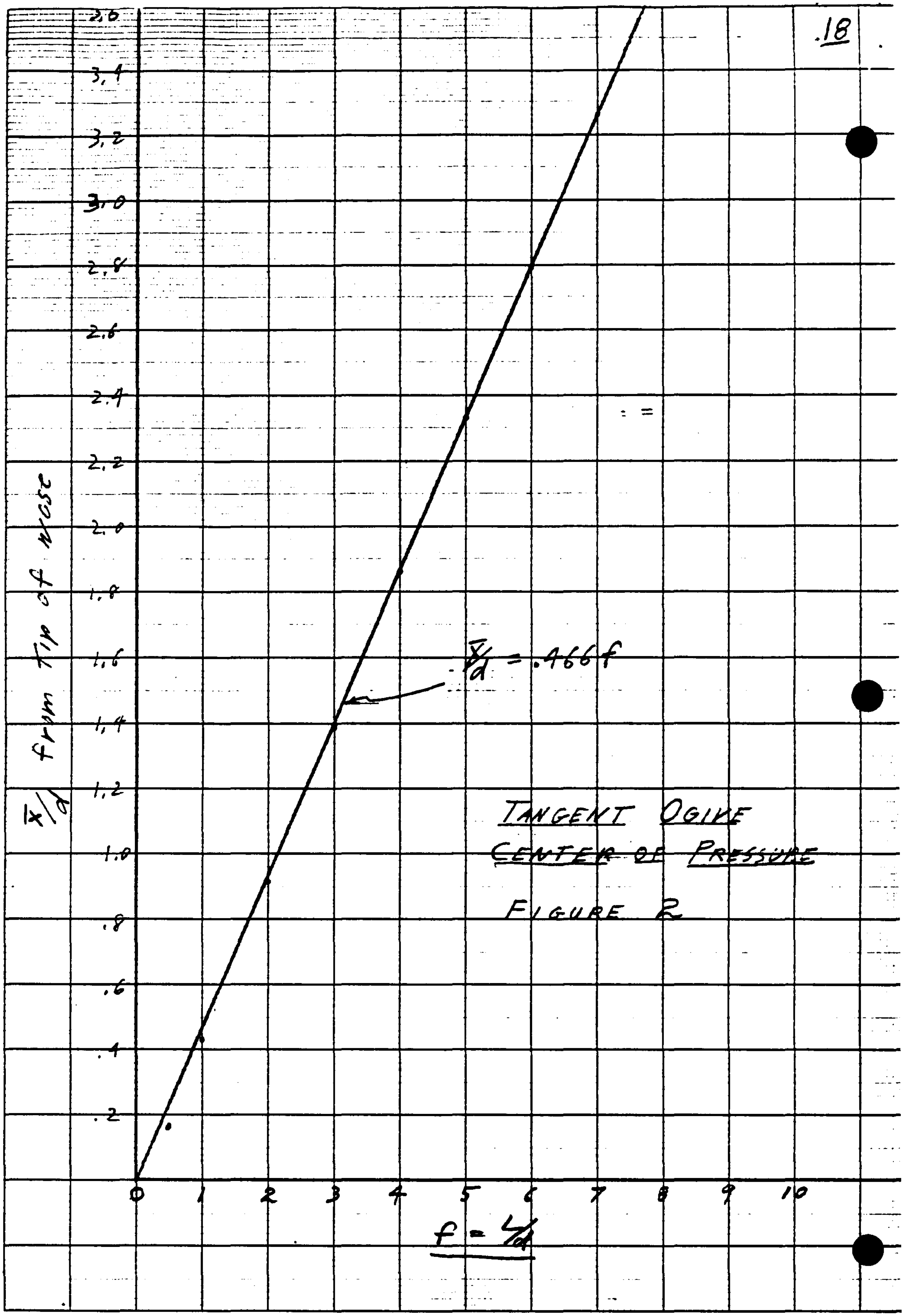
Equation 37 is solved numerically and plotted in figure 2 . A computer program, as listed on the next page, was used to do the calculation with extreme accuracy.

As can be seen in figure 2 , the resultant curve is very nearly a straight line. Thus; equation 37 may be approximated very well be the equation of the straight line as long as f is greater than one (1) .

$$\bar{X}/d = .466\,f = .466\,L/d \qquad (38)$$

dividing both sides of equation 38 by d ;

$$\boxed{\bar{X}_N = .466\,L} \quad \text{OGIVE} \qquad (39)$$



TANGENT OGIVE
CENTER OF PRESSURE

FIGURE 2

```
C       CENTER OF PRESSURE OF AN OGIVE
        DOUBLE PRECISION A,B,C,D,E,F,G,H,XCP
        WRITE(6,2)
      2 FORMAT(13H1    F      X/D)
        DO 10 I=1,10
        F=I
        A=F*F
        B=A+.25
        C=A-.25
        D=B*B
        E=A*F
        G=F/B
        H=DATAN(DABS(G/DSQRT(1.-G*G)))
        XCP = F + 4.*D*(C*H - F) + 4.*E/3.
     10 WRITE(6,1)F,XCP
      1 FORMAT(1H ,F5.0,F9.3)
        STOP
        END
```

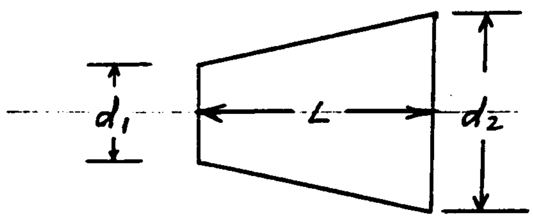
OUTPUT

F	X/D
1.	0.430
2.	0.914
3.	1.387
4.	1.857
5.	2.326
6.	2.794
7.	3.261
8.	3.729
9.	4.196
10.	4.663

Cylindrical Body

Since $C_{N\alpha} = 0$ for a cylindrical body, calculation of $\bar{X}$ is not necessary.

Conical Sholder



The volume of a conical frustrum is;

$$v = \frac{\pi L}{12}\left(d_1^2 + d_1 d_2 + d_2^2\right) \qquad (40)$$

or

$$v = \frac{L}{3}\left(S_1 + S_2 \frac{d_1}{d_2} + S_2\right)$$

or

$$v = \frac{L S_2}{3}\left(\frac{S_1}{S_2} + \frac{d_1}{d_2} + 1\right)$$

But, since

$$S_2 = S(L)$$

then,

$$\frac{v}{S(L)} = \frac{L}{3}\left(\frac{S_1}{S_2} + \frac{d_1}{d_2} + 1\right) \qquad (41)$$

Also,

$$\frac{S_1}{S_2} = \left(\frac{d_1}{d_2}\right)^2.$$

thus,

$$\frac{v}{S(L)} = \frac{L}{3}\left[1 + \frac{d_1}{d_2} + \left(\frac{d_1}{d_2}\right)^2\right] \qquad (42)$$

Substituting equation 42 in equation 29;

$$\bar{X} = \frac{L - \frac{L}{3}\left[1 + \frac{d_1}{d_2} + \left(\frac{d_1}{d_2}\right)^2\right]}{1 - \frac{S(0)}{S(L)}} \qquad (43)$$

Again; noting that

$$\frac{S(0)}{S(L)} = \frac{S_1}{S_2} = \left(\frac{d_1}{d_2}\right)^2$$

and expanding;

$$\bar{X} = \frac{L}{3}\left[\frac{3 - 1 - \frac{d_1}{d_2} - \left(\frac{d_1}{d_2}\right)^2}{1 - (d_1/d_2)^2}\right]$$

$$= \frac{L}{3}\left[\frac{2 - d_1/d_2 - (d_1/d_2)^2}{1 - (d_1/d_2)^2}\right]$$

$$= \frac{L}{3}\left[\frac{1 - (d_1/d_2)^2}{1 - (d_1/d_2)^2} + \frac{1 - d_1/d_2}{1 - (d_1/d_2)^2}\right]$$

$$\boxed{\bar{X}_{cs \text{ or } cB} = \frac{L}{3}\left[1 + \frac{1 - d_1/d_2}{1 - (d_1/d_2)^2}\right]} \qquad (44)$$

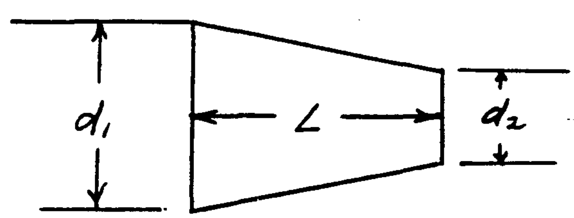
from front of sholder or Boattail

$$\bar{X}_{[illegible]} = \frac{L}{3}\left(1 + \frac{1}{1 + d_1/d_2}\right)$$

as per [illegible] 8, [illegible]

Conical Boattail

Since no distinction as to direction of the conical frustrum was made in deriving equation 44, it holds true also for a frustrum with the dimensions shown;



FIN AERODYNAMICS DERIVATIONS

Normal Force Coefficient Slope

From Reference 1, by a theory of Diederich, C_{N_α} of a finite flat plate is given by;

$$C_{N_\alpha} = \frac{C_{N_{\alpha_0}} F \left(\frac{A_f}{A}\right) \cos\sigma}{2 + F\sqrt{1 + 4/F^2}} \quad (45)$$

where:

$C_{N_{\alpha_0}}$ = Normal force coefficient slope of a two dimensional airfoil.
F = Diederich's correlation parameter
A_f = Area of one fin

According to Diederich;

$$F \equiv \frac{AR}{\frac{1}{2\pi} C_{N_{\alpha_0}} \cos\sigma} \quad (46)$$

By the thin airfoil theory of potential flow;

$$C_{N_{\alpha_0}} = 2\pi \quad (47)$$

Thus;

$$F \equiv \frac{AR}{\cos\sigma} \quad (48)$$

Substituting equations 47 and 48 into 45;

$$C_{N_\alpha} = \frac{2\pi AR \left(A_f/A\right)}{2 + \frac{AR}{\cos\sigma}\sqrt{1 + \frac{4\cos^2\sigma}{AR^2}}} \quad (49)$$

Simplifying;

$$C_{N_\alpha} = \frac{2\pi A\!\!R\,(A_f/A)}{2 + \sqrt{4 + \left(\frac{A\!\!R}{\cos\sigma}\right)^2}} \qquad (50)$$

This is C_{N_α} for a single fin.

A typical fin has the geometry shown in figure 3. All fins can be idealized into a fin or a set of fins having straight line edges as shown in figure 3 .

By definition;

$$A\!\!R = \frac{2s^2}{A_f} \qquad (51)$$

Also;

$$A = \frac{\pi d^2}{4} \qquad (52)$$

Substituting 51 and 52 into the numerator of equation 50;

$$2\pi A\!\!R\,(A_f/A) = 2\pi\left(\frac{2s^2}{A_f}\right)\left(\frac{A_f}{\frac{\pi}{4}d^2}\right)$$

$$2\pi A\!\!R\,(A_f/A) = 16\left(\frac{s}{d}\right)^2 \qquad (53)$$

By trigonometric definition;

$$\cos\sigma = s/l \qquad (54)$$

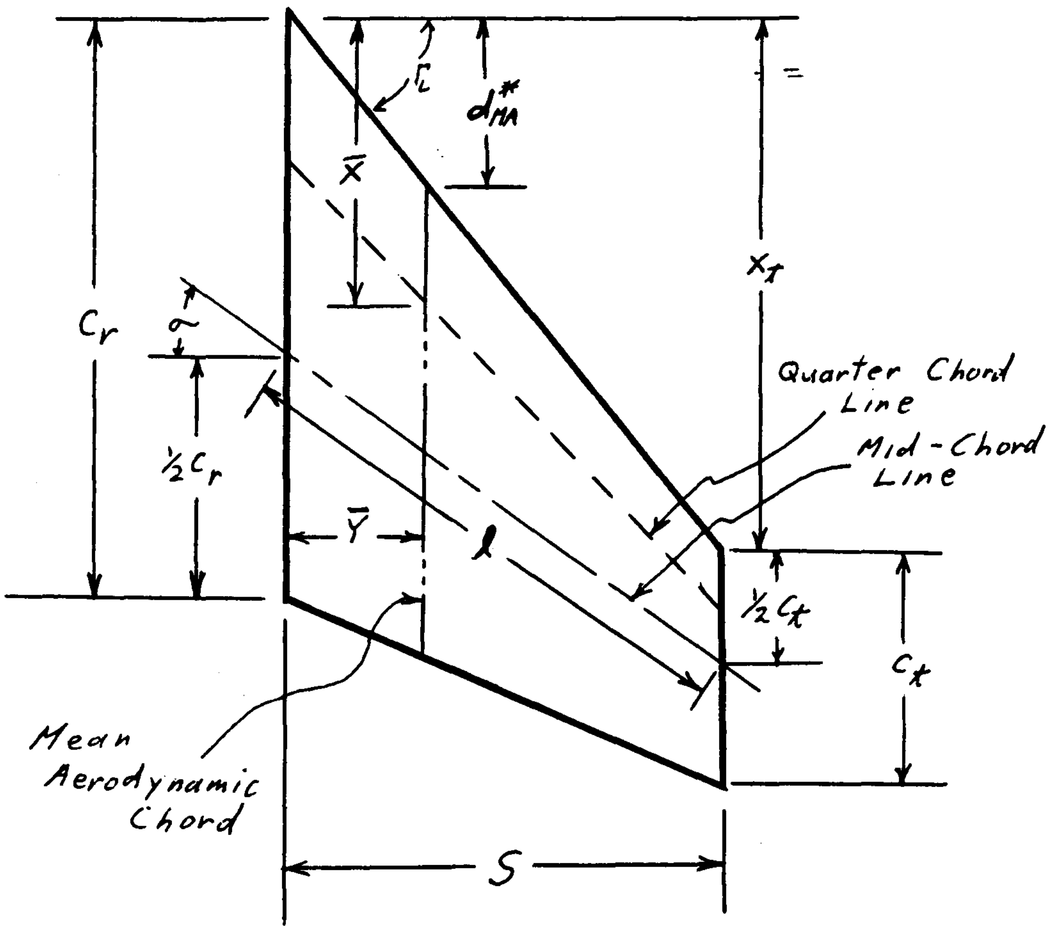


FIGURE 3 - FIN GEOMETRY

Then;

$$\frac{A\!R}{\cos\sigma} = \frac{2s^{\not 2}}{A_f}\frac{\ell}{\not s} = \frac{2\ell s}{A_f}$$

But; from geometry;

$$A_f = \left(\frac{C_r + C_t}{2}\right)s$$

Therefore;

$$\frac{A\!R}{\cos\sigma} = \frac{2\ell \not s}{\left(\frac{C_r + C_t}{2}\right)\not s}$$

$$\frac{A\!R}{\cos\sigma} = \frac{4\ell}{C_r + C_t} \qquad (55)$$

Substituting 55 into the denominator of 50;

$$2 + \sqrt{4 + \left(\frac{A\!R}{\cos\sigma}\right)^2} = 2 + \sqrt{4 + \left(\frac{4\ell}{C_r + C_t}\right)^2}$$

$$= 2 + \sqrt{4 + 4\left(\frac{2\ell}{C_r + C_t}\right)^2}$$

$$2 + \sqrt{4 + \left(\frac{A\!R}{\cos\sigma}\right)^2} = 2 + 2\sqrt{1 + \left(\frac{2\ell}{C_r + C_t}\right)^2} \qquad (56)$$

Substituting equation 53 and 56 into 50;

$$C_{N_\alpha} = \frac{16\left(\frac{s}{d}\right)^2}{2 + 2\sqrt{1 + \left(\frac{2\ell}{C_r + C_t}\right)^2}}$$

Simplifying;

$$C_{N_\alpha} = \frac{8\left(\frac{s}{d}\right)^2}{1+\sqrt{1+\left(\frac{2l}{C_r+C_t}\right)^2}} \quad (57)$$

Equation 57 gives C_{N_α} for a single fin. A four fin rocket, having two fins in the plane normal to the plane of the angle of attack (see figure 4a) has the C_{N_α} of;

$$\left(C_{N_\alpha}\right)_F = \frac{16\left(\frac{s}{d}\right)^2}{1+\sqrt{1+\left(\frac{2l}{C_r+C_t}\right)^2}} \quad (58)$$

A three finned rocket has its fins spaced 120° apart. Assuming that the 3 finned rocket flys with one fin in the plane of the angle of attack, with $(C_{N_\alpha})_1 = C_{N_\alpha}$ of one fin; (See figure 4b)

$$C_{N_\alpha} = 2\left(C_{N_\alpha}\right)_1 \cos 30^\circ$$

$$= 2\left(C_{N_\alpha}\right)_1 \frac{\sqrt{3}}{2}$$

$$C_{N_\alpha} = \sqrt{3}\left(C_{N_\alpha}\right)_1$$

Thus;

$$\left(C_{N_\alpha}\right)_F = \frac{8\sqrt{3}\left(\frac{s}{d}\right)^2}{1+\sqrt{1+\left(\frac{2l}{C_r+C_t}\right)^2}}$$

or

see addendum 2

$$\left(C_{N_\alpha}\right)_F = \frac{12 \;\sout{13.85}\left(\frac{s}{d}\right)^2}{1+\sqrt{1+\left(\frac{2l}{C_r+C_t}\right)^2}} \quad (59)$$

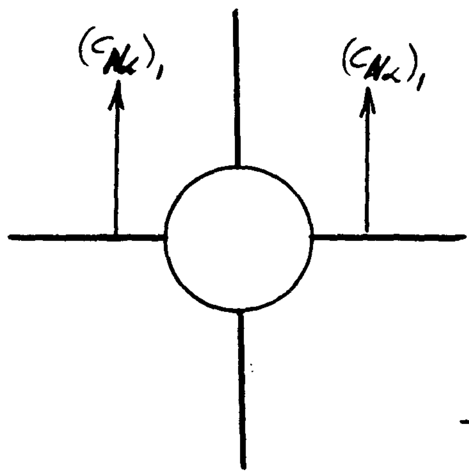


FIGURE 4a
FOUR FINS

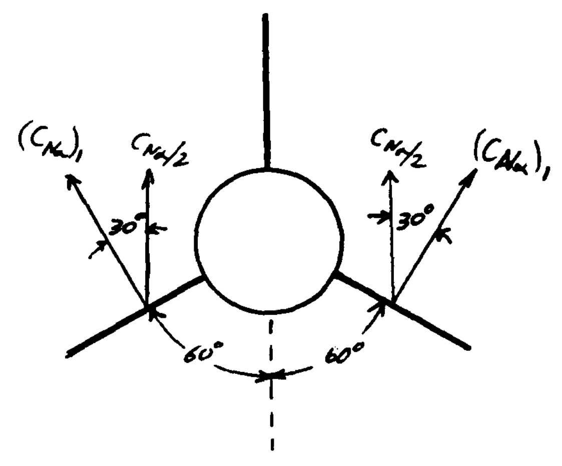


FIGURE 4b
THREE FINS

FIN AERODYNAMICS DERIVATIONS

Center of Pressure

From the potential theory of subsonic flow, the center of pressure of a two dimensional airfoil is located at 1/4 the length of its chord from its leading edge. Thus; on a three dimensional fin, the center of pressure should be located along the quarter chord line.

By definition, the spanwise center of pressure is located along the mean aerodynamic chord. Therefore, by the above argument, the fin center of pressure is located at the intersection of the quarter chord line and the mean aerodynamic chord. (See figure 3)

It remains to determine the length position of the mean aerodynamic chord.

By definition, the mean aerodynamic chord is;

$$C_{MA} = \frac{1}{A_f}\int_0^S c^2\,dy \qquad (60)$$

where;(See figure 5)

A_f = Area of one fin.
S = Semispan of one fin.
c = Generalized chord.
y = Spanwise coordinate.

The generalized Chord is a function of the span. To find this function, a proportionality relation is set up. (See figure 6)

$$\frac{C_r}{L^*} = \frac{C}{L^*-y} = \frac{C_t}{L^*-S} \qquad (61)$$

From the first two terms;

$$C = \frac{C_r\,(L^*-y)}{L^*}$$

$$C = C_r - \frac{y}{L^*}C_r \qquad (62)$$

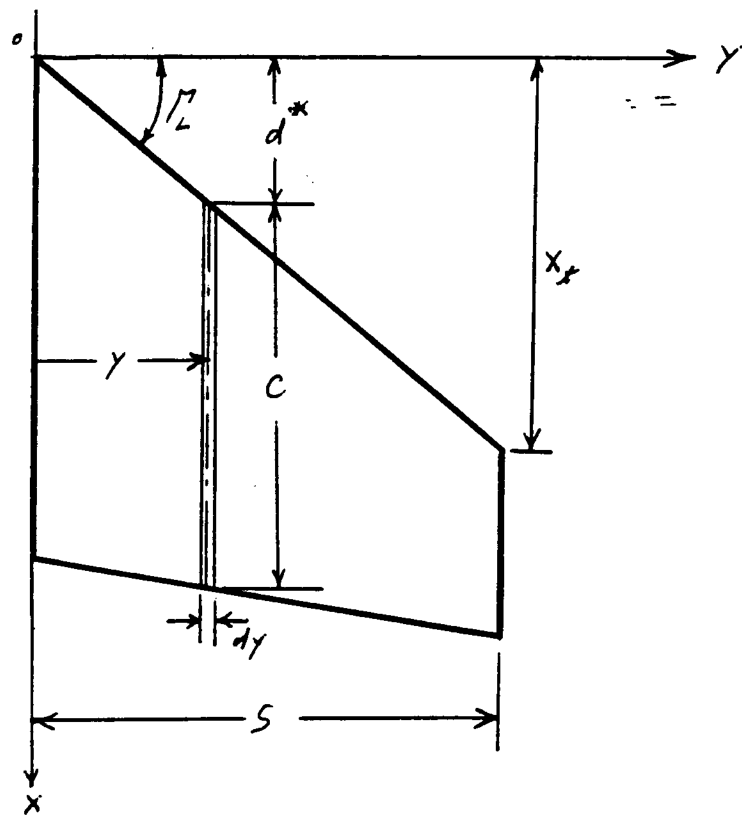


FIGURE 5

COORDINATE SYSTEM FOR THE DETERMINATION OF THE MEAN AERODYNAMIC CHORD

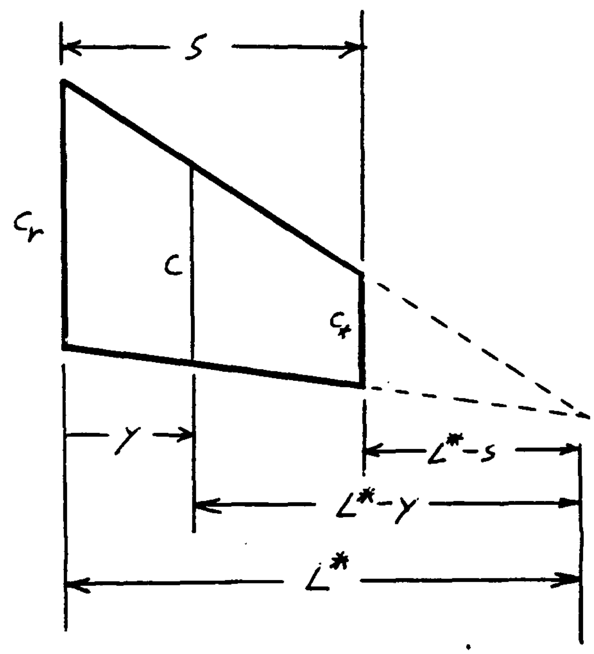


FIGURE 6

TRIANGLE OF PROPORTIONALITY
FOR THE DETERMINATION OF
THE GENERAL CHORD LENGTH

From the first and last terms

$$c_t L^* = c_r L^* - c_r s$$

or

$$L^* (c_r - c_t) = c_r s$$

or

$$L^* (c_r - c_t) = L^* c_r (1 - \lambda) = c_r s$$

thus

$$L^* = \frac{s}{1 - \lambda} \qquad (63)$$

substituting 63 into 62

$$c = c_r \left[1 + \left(\frac{\lambda - 1}{s} \right) y \right] \qquad (64)$$

substituting 64 into 60

$$c_{MA} = \frac{1}{A_f} \int_0^s c_r^2 \left[1 + \left(\frac{\lambda - 1}{s} \right) y \right]^2 dy$$

Expanding;

$$c_{MA} = \frac{c_r^2}{A_f} \int_0^s \left[1 + 2 \left(\frac{\lambda - 1}{s} \right) y + \left(\frac{\lambda - 1}{s} \right)^2 y^2 \right] dy$$

Let; $k = \frac{\lambda - 1}{s}$

$$C_{MA} = \frac{C_r^2}{A_f}\int_0^s \left[1 + 2ky + k^2y^2\right] dy \qquad (65)$$

Performing the integration;

$$C_{MA} = \frac{C_r^2}{A_f}\left\{\int_0^s dy + 2k\int_0^s y\,dy + k^2\int_0^s y^2\,dy\right\}$$

$$= \frac{C_r^2}{A_f}\left\{\left[y\right]_0^s + 2k\left[\frac{y^2}{2}\right]_0^s + k^2\left[\tfrac{1}{3}y^3\right]_0^s\right\}$$

$$C_{MA} = \frac{C_r^2}{A_f}\left[s + ks^2 + \tfrac{1}{3}k^2s^3\right] \qquad (66)$$

Substitute 66 in 65 and simplifying

$$C_{MA} = \frac{C_r^2 s}{A_f}\left[1 + ks + \tfrac{1}{3}k^2s^2\right]$$

$$= \frac{C_r^2 s}{A_f}\left[1 + (\lambda - 1) + \tfrac{1}{3}(\lambda - 1)^2\right]$$

$$= \frac{C_r^2 s}{A_f}\left[\lambda + \tfrac{1}{3}(\lambda^2 - 2\lambda + 1)\right]$$

$$C_{MA} = \frac{1}{3}\,\frac{C_r^2 s}{A_f}\left[\lambda^2 + \lambda + 1\right] \qquad (67)$$

But; by geometry

$$A_f = \tfrac{1}{2}(C_r + C_t)\,s \qquad (68)$$

Thus, Substituting 68 in 67;

$$C_{MA} = \frac{2}{3}\frac{C_r^2}{C_r+C_t}\left[\lambda^2+\lambda+1\right]$$

$$= \frac{2}{3}\frac{1}{C_r+C_t}\left[C_t^2+C_rC_t+C_r^2\right]$$

$$= \frac{2}{3}\frac{1}{C_r+C_t}\left[(C_r+C_t)^2-C_rC_t\right]$$

$$C_{MA} = \frac{2}{3}\left[C_r+C_t-\frac{C_rC_t}{C_r+C_t}\right] \qquad (69)$$

It is now necessary to find the spanwise position of C_{MA}. This is done by equating equation 69 and 64 and solving for $\bar{Y}$.

$$\frac{2}{3}\left[C_r+C_t-\frac{C_rC_t}{C_r+C_t}\right] = C_r\left[1+\left(\frac{\lambda-1}{S}\right)\bar{Y}\right]$$

$$= C_r + \left(\frac{C_t-C_r}{S}\right)\bar{Y}$$

Thus;

$$\bar{Y} = \left[\frac{2}{3}C_r+\frac{2}{3}C_t-\frac{2C_rC_t}{3(C_r+C_t)}-C_r\right]\frac{S}{C_t-C_r}$$

$$\bar{Y} = \frac{S}{3(C_t - C_r)}\left[2C_t - C_r - \frac{2C_rC_t}{C_r + C_t}\right]$$

$$= \frac{S}{3(C_t-C_r)(C_r+C_t)}\left[\cancel{2C_rC_t} + 2C_t^2 - C_r^2 - C_rC_t - \cancel{2C_rC_t}\right]$$

$$= \frac{S}{3(C_t-C_r)(C_r+C_t)}\left[2C_t^2 - C_rC_t - C_r^2\right]$$

$$= \frac{S}{3}\left[\frac{(2C_r+C_t)\cancel{(C_t-C_r)}}{\cancel{(C_t-C_r)}(C_r+C_t)}\right]$$

$$\bar{Y} = \frac{S}{3}\,\frac{(C_r+2C_t)}{(C_r+C_t)} \qquad (70)$$

By trigonometry; (See figure 5)

$$d^*_{MA} = \bar{Y}\tan\Gamma_L \qquad (71)$$

And,

$$\tan\Gamma_L = \frac{x_t}{S} \qquad (72)$$

Thus;

$$d^*_{MA} = \frac{\bar{Y}}{S}\,x_t \qquad (73)$$

Substituting 73 into 70

$$d^*_{MA} = \frac{X_t}{3}\,\frac{(C_r+2C_t)}{(C_r+C_t)} \qquad (74)$$

From the argument at the beginning of this section;

$$\bar{X} = d + \tfrac{1}{4}\,C_{MA} \qquad (75)$$

Substituting equations 74 and 69 into 75

$$\bar{X}_F = \frac{X_t}{3}\,\frac{(C_r+2C_t)}{(C_r+C_t)} + \tfrac{1}{6}\left[C_r + C_t - \frac{C_r C_t}{C_r+C_t}\right] \qquad (76a)$$

This $\bar{X}$ is from the leading edge of the root chord. To get the center of pressure of the fins from the nose tip, X_F must be added to $\bar{X}_F$.

X_F = Distance from nose tip to leading edge of fin root chord.

$$(\bar{X})_{T(B)} = X_F + \bar{X}_F \qquad (76b)$$

INTERFERENCE EFFECTS

The major interference effects encountered on any rocket are the change of lift of the fin alone when it is brought into the presence of the body and the change of lift on the body between the fins. Reference 3 discusses these effects in detail. They are handled by the use of correction factors which are applied to the fins alone . The values of these factors are shown in figure 7. The plots of interest are underlined in red. In this figure, "S" is actually $(S+r_t)$ in my nomenclature.

$K_{T(B)}$ = Correction factor for the fins in the presence of the body

$K_{B(T)}$ = Correction factor for the body in the presence of the fins.

As can be seen in figure 7, the value of $K_{T(B)}$ is considerably greater than that for $K_{B(T)}$ in the range of $r_t/(S+r_t)$ in which most model rockets fall $(<.4)$. Thus, it is a conservative and reasonable approximation to do two things to simplify the interference calculations.

1) Approximate the $K_{T(B)}$ curve by a straight line. (red line on figure 7)
2) Neglect the influence of $K_{B(T)}$.

In this way;

$$K_{T(B)} = 1 + \frac{r_t}{S+r_t} \qquad (77)$$

Thus;

$$(C_{N\alpha})_{T(B)} = K_{T(B)}\,(C_{N\alpha})_{\text{fins Alone}} \qquad (78)$$

where; $(C_{N\alpha})_{\text{fins Alone}}$ is obtained from equation 58 or equation 59; and $K_{T(B)}$ comes from equation 77.

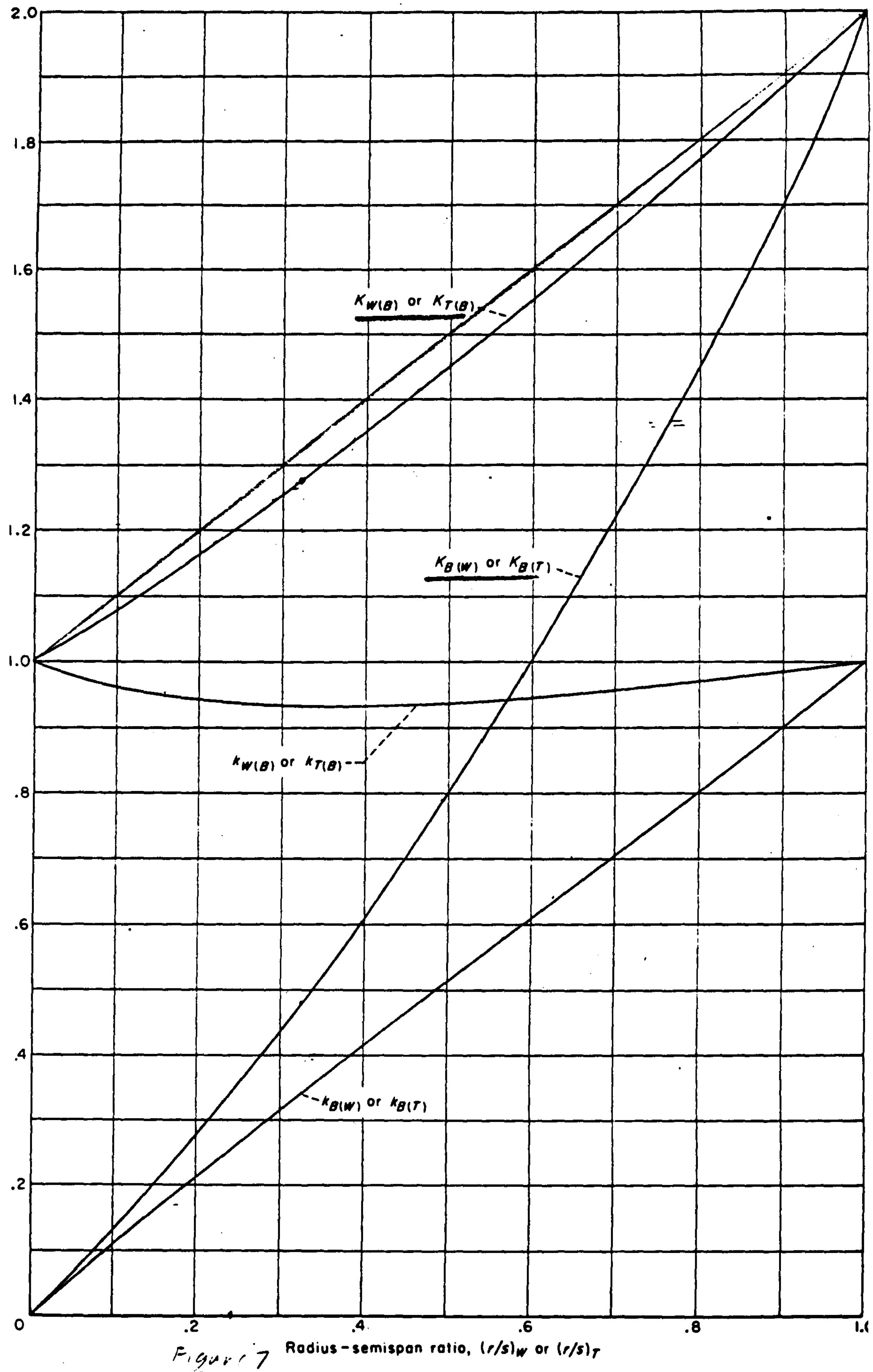


Figure 7.—Values of lift ratios based on slender-body theory.

COMBINATION CALCULATIONS

The total vehicle $C_{N\alpha}$ is the sum of the $C_{N\alpha}$'s of the individual portions;

$$C_{N\alpha} = (C_{N\alpha})_N + (C_{N\alpha})_{T(B)} + (C_{N\alpha})_{CS} + (C_{N\alpha})_{CB} \quad (79)$$

The center of pressure is determined by a moment balance about the nose of the rocket.

$$\bar{X} = \frac{(C_{N\alpha})_N \bar{X}_N + (C_{N\alpha})_{T(B)} \bar{X}_{T(B)} + (C_{N\alpha})_{CS} \bar{X}_{CS} + (C_{N\alpha})_{CB} \bar{X}_{CB}}{C_{N\alpha}} \quad (80)$$

Of course, if there are more than one conical shoulder, conical boattail, and/or fins; these are also included in equations 78 and 79.

EXPERIMENTAL VERIFICATION

Ideally, the use of a windtunnel is required to accurately determine the aerodynamic coefficients of a rocket. Since none is available for model rockets (at least available to the author), it was decided to use a series of flight tests to determine the center of pressure.

The technique is to use a rocket which has a center of gravity that can be changed from flight to flight. Once the rocket design has been analysed using the derived equations, the rocket is flown in a series of flights. The C.G. is progressively moved back toward the calculated C.P. position. The C. G. position at which the rocket becomes unstable is taken as the true C.P. position.

The original experimental data, drawing of rocket, and theorical calculations are shown in their original form on the following pages. In addition, the data from windtunnel tests on a full size sounding rocket (Aerobee 350) are presented along with the theoretical calculations.

Conclusions (GENERAL)

The equations as derived and simplified predict the center of pressure of a general rocket configuration to within one (1.) percent. And, as such, are useful in the design and analysis of model rockets.

Compilation of Equations

For ease of use, the equations as derived are compiled on the last few pages of this report.

EXPERIMENTAL VERIFICATION

TESTBED II FLIGHT DATA

Flight	C.G. Location	Observation of Flight
1	14.7"	No Wobbling or weathercocking in Low wind
2	15.7"	No Wobbling or weathercocking in Low wind
3	16.25"	Slight Wobbling; Weathercocked in Moderate Wind.
4	16.25"	No Wobbling or weathercocking in Low wind
5	16.5"	Bad Wobbling; No Weathercocking in Low Wind
6	16.5"	Bad Wobbling; No Weathercocking in Low Wind.
7	16.7"	Completely Unstable; Looped
8	16.7"	Completely Unstable; Violent Swerving
9	16.6"	Completely Unstable; Violent Swerving
10	16.6"	Completely Unstable; Violent Swerving
11	16.55"	Unstable; Swerved badly but flew roughly vertically
12	16.55"	Badly Unstable; Zigzagged horizontally
13	16.55"	Unstable; Spiralling flight with large Coning
14	16.5"	Bad Wobbling; No Weathercocking in moderate wind
15	16.55"	Unstable; Swerved badly but flew roughly vertically

Notes:

1. All flights were with Estes Industries A.8-3 engines.
2. C.G. Location is the average of the C.G. locations before and after flight (To the nearest 5/100")
3. C.G. Location was determined by suspending the model by a thread.

Discussion of Results

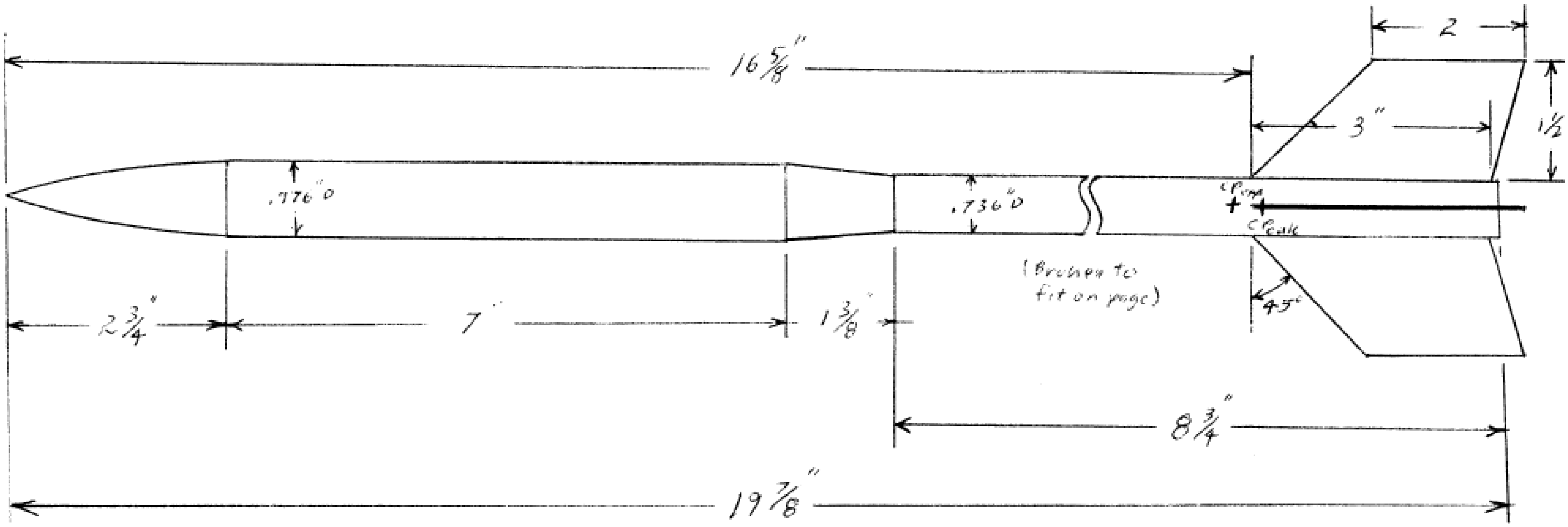
Both the flight and weathercocking characteristics of the flights having the C.G. at 16.5" indicate that the rocket is very close to the neutral stability point. Also, the flights having a C.G. location ahead of 16.5" are stable; while the flights with the C.G. behind 16.5" are unstable.

Thus, <u>the center of pressure of the testbed II model rocket is 16.5" from the nose</u>. This gives a percent error of;

$$\% \text{ error} = \left[\frac{16.7 - 16.5}{16.5}\right] 100$$

$$= \frac{20}{16.5}$$

$$\% \text{ error} = 1.21\%$$



TESTBED II MODEL ROCKET

Testbed II Calculations

Nose

Shape: Ogive

$(C_{N\alpha})_N = 2.$

$\bar{X}_N = .466\,(2.75)$

$\bar{X}_N = 1.28''$

Fins (4)

$$(C_{N\alpha})_F = \frac{16\,(s/d)^2}{1 + \sqrt{1 + \left(\frac{2\ell}{C_r + C_t}\right)^2}}$$

$$= \frac{16\,(1.5/.976)^2}{1 + \sqrt{1 + \left(\frac{2 \times 1.81}{3 + 2}\right)^2}}$$

$$= \frac{16\,(1.538)^2}{1 + \sqrt{1 + (.724)^2}} = \frac{16\,(2.37)}{1 + \sqrt{1.515}}$$

$$= \frac{37.9}{2.23}$$

$(C_{N\alpha})_F = 17.0$

$$\bar{X}_F = \frac{S(C_r + 2C_x)}{3(C_r + C_x)} + \frac{1}{6}\left[C_r + C_x - \frac{C_r C_x}{C_r + C_x}\right]$$

$$= \frac{1.5(3+4)}{3(3+2)} + \frac{1}{6}\left[3+2 - \frac{3(2)}{3+2}\right]$$

$$= \frac{.5(7)}{5} + \frac{1}{6}\left[5 - \frac{6}{5}\right]$$

$$= .7 + \frac{1}{6}(5 - 1.2) = .7 + \frac{1}{6}(3.8)$$

$$= .7 + .633$$

$$\underline{\bar{X}_F = 1.333''}$$

$$\bar{X}_{T(B)} = 1.333 + 16.625$$

$$\underline{\bar{X}_{T(B)} = 17.958''}$$

<u>Interference</u>

$$K_{T(B)} = 1 + \frac{r_x}{S + r_x} = 1 + \frac{.368}{1.5 + .368}$$

$$= 1 + \frac{.368}{1.868} = 1 + .197$$

$$\underline{K_{T(B)} = 1.197}$$

Conical Boattail

$$(C_{N\alpha})_{CB} = \frac{8}{\pi (d^2)} \left[S_2 - S_1 \right]$$

$$= \frac{2\,8}{\pi(.976)^2} \left[\frac{\pi}{4}(.736)^2 - \frac{\pi}{4}(.976)^2 \right]$$

$$= 2\left[\left(\frac{.736}{.976}\right)^2 - 1 \right] = 2\left[(.754)^2 - 1 \right]$$

$$= 2\,(.57 - 1) = 2\,(-.43)$$

$$\underline{(C_{N\alpha})_{CB} = -.86}$$

$$\bar{x}_{CB_0} = \frac{L}{3}\left[1 + \frac{1 - d_1/d_2}{1 - (d_1/d_2)^2} \right]$$

$$= \frac{1.375}{3}\left[1 + \frac{1 - .754}{1 - .57} \right]$$

$$= .458\left[1 + \left(\frac{.246}{.43}\right) \right] = .458(1.572)$$

$$\underline{\bar{X}_{CB_0} = 0.72''}$$

$$X_{CB} = 9.75''$$

$$\therefore \; \bar{X}_{CB} = 9.75 + .72 = \underline{10.47}''$$

Total Values

$$C_{N_\alpha} = 2 + 20.3 - .86$$

$$C_{N_\alpha} = 21.44$$

$$\bar{X} = \frac{2(1.28) + 20.3(17.96) + (-.86)(10.47)}{21.44}$$

$$= \frac{2.56 + 364.55 - 9.01}{21.44} = \frac{358.1}{21.44}$$

$$\bar{X} = 16.7''$$

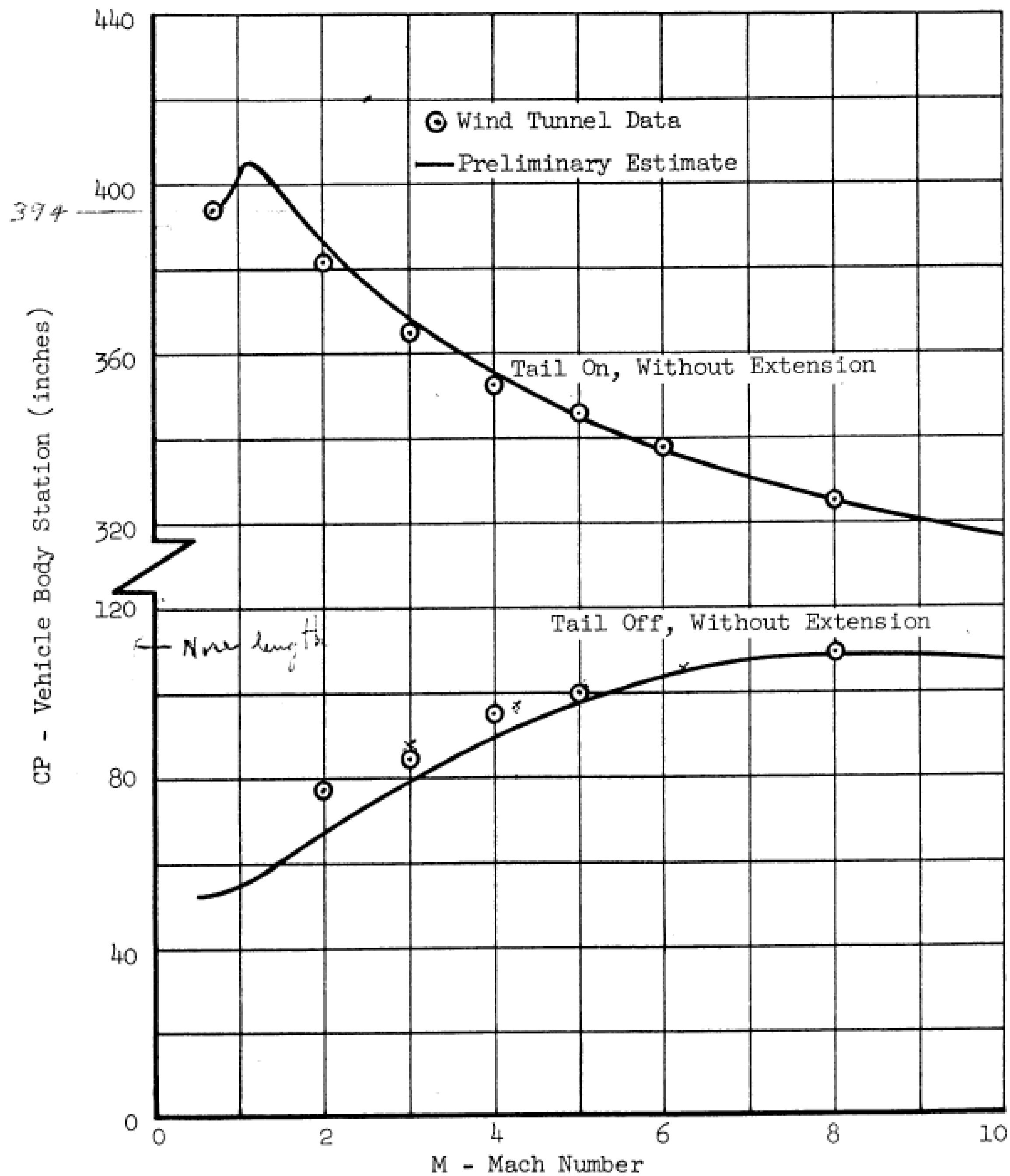
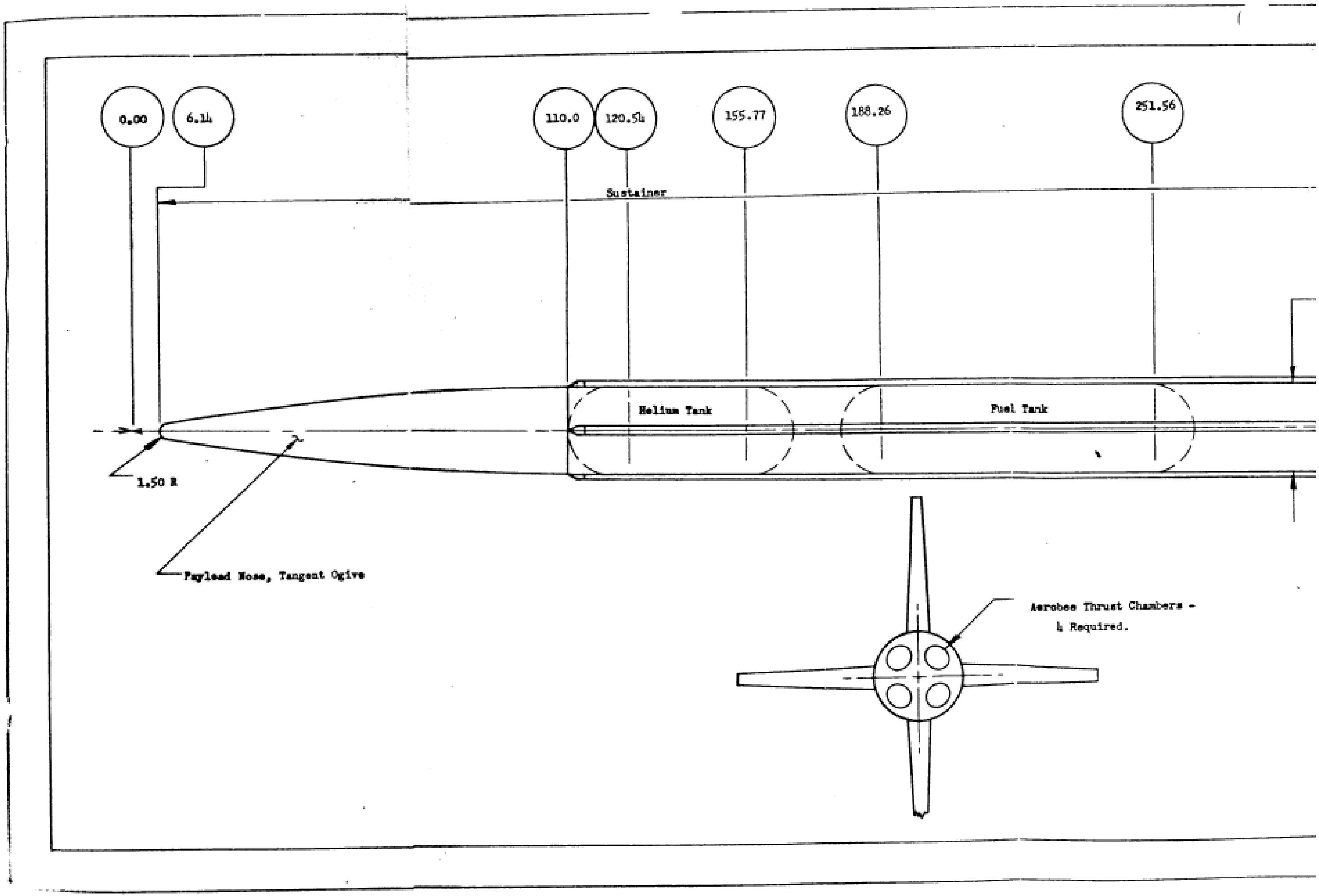


FIGURE 22. Aerobee 350, Center of Pressure Versus Mach Number


0.00
6.14
110.0
120.54
155.77
188.26
251.56
Sustainer
Helium Tank
Fuel Tank
1.50 R
Payload Nose, Tangent Ogive
Aerobee Thrust Chambers -
4 Required.

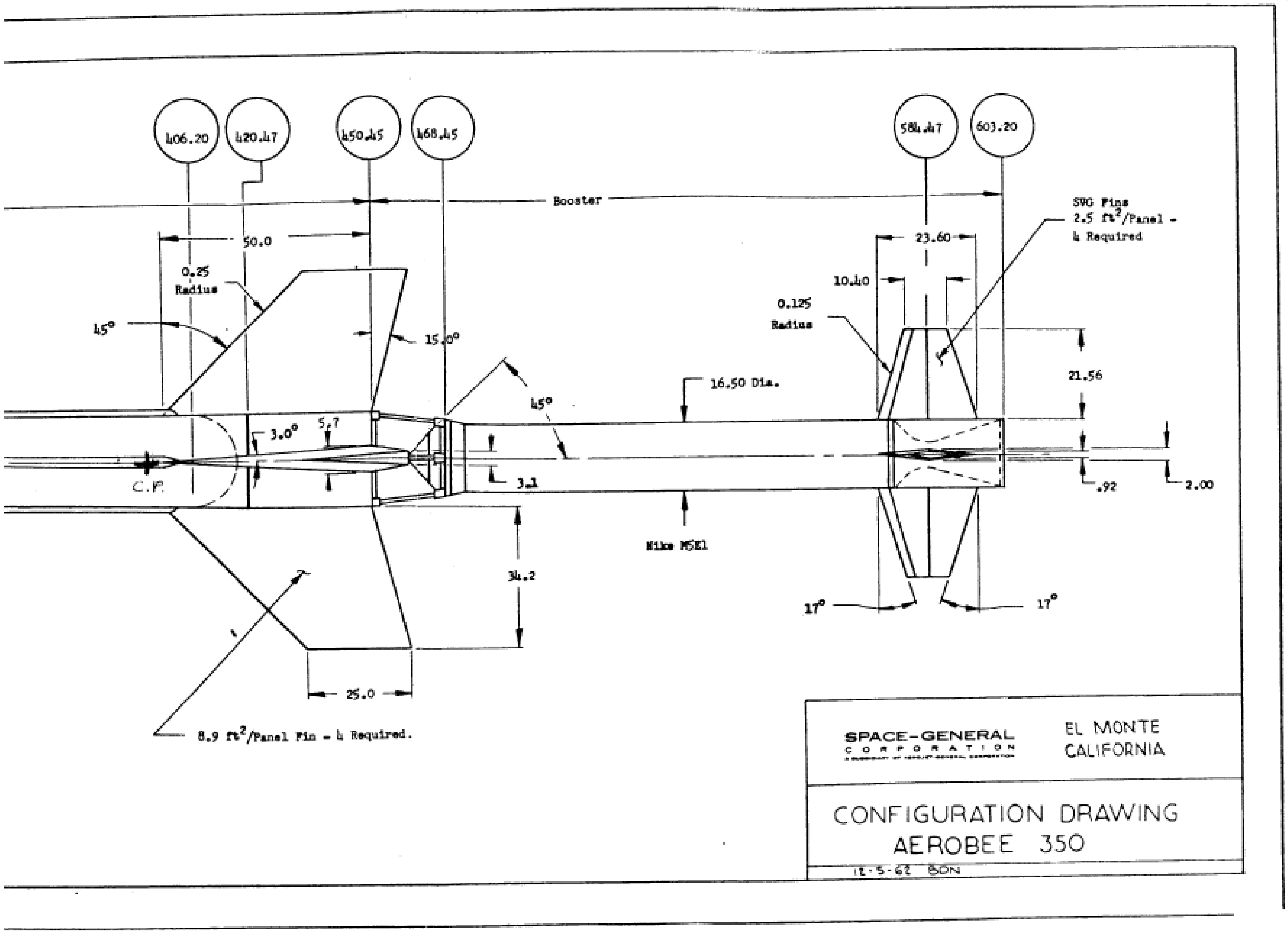


FIGURE 1 Vehicle Configuration Drawing

Aerobee 350 Calculations

See Configuration Drawing

Nose

Shape. - Ogive

$$\underline{(C_{N\alpha})_N = 2.}$$

$$\bar{X}_N = .466\,(110.) = \underline{51.3''}$$

FINS Four (4)

$$(C_{N\alpha})_F = \frac{16\,(s/d)^2}{1 + \sqrt{1 + \left(\frac{2\ell}{C_r + C_t}\right)^2}}$$

$$(C_{N\alpha})_F = \frac{16\,(34.2/22)^2}{1 + \sqrt{1 + \left(\frac{2 \times 39.7}{50 + 25}\right)^2}}$$

ℓ found by measuring enlarged fin drawing.

$$(C_{N\alpha})_F = \frac{16\,(1.553)^2}{1 + \sqrt{1 + (1.509)^2}} = \frac{16\,(2.42)}{1 + \sqrt{2.12}}$$

$$= \frac{38.7}{2.456}$$

$$\underline{(C_{N\alpha})_F = 15.7}$$

$$\bar{X}_F = \frac{s}{3}\,\frac{(C_r+2C_t)}{(C_r+C_t)} + \frac{1}{6}\left(C_r+C_t - \frac{C_rC_t}{C_r+C_t}\right)$$

$$= \frac{34.2\,(50+50)}{3\,(50+25)} + \frac{1}{6}\left(50+25 - \frac{50(25)}{50+25}\right)$$

$$= \frac{3420}{225} + \frac{1}{6}\left(75 - \frac{1250}{75}\right)$$

$$= 15.2 + \tfrac{1}{6}(75 - 16.68)$$

$$= 15.2 + \tfrac{1}{6}(58.32) = 15.2 + 9.72$$

$$\underline{\bar{X}_F = 24.92''}$$

$$X_F = 400.45$$

$$\therefore \underline{(\bar{X})_{T(B)} = 425.37''}$$

$$K_{T(B)} = ** = 1 + \frac{r_t}{r_t+s}$$

$$= 1 + \frac{11}{11+34.2} = 1 + \frac{11}{45.2}$$

$$= 1 + .243$$

$$K_{T(B)} = 1.243$$

$$\therefore (C_{N\alpha})_{T(B)} = 1.243(15.7)$$

$$\underline{(C_{N\alpha})_{T(B)} = 19.5}$$

Total Values

$$C_{N_\alpha} = 2 + 19.5$$

$$C_{N_\alpha} = 21.5$$

$$\bar{X} = \frac{2(51.3) + 19.5(425.45)}{21.5}$$

$$= \frac{102.6 + 8296.3}{21.5} = \frac{8398.9}{21.5}$$

$$\bar{X} = 391''$$

As read from windtunnel data at Mach 0.6

$$\bar{X}_{exp} = 394''$$

The value at Mach 0.6 is probably very slightly high, but is very close to the value at Mach zero.

The percent error is then;

$$\Delta\% = \left[\frac{394-391}{391}\right]100 = \frac{300}{391}$$

$$\%\ error = 0.77\%$$

REFERENCES

1) Shapiro, A. H.; The Dynamics and Thermodynamics of Compressible Fluid Flow, Vol. 1; Ronald; New York; 1953.

2) Mayo, E. E.; Cone Cylinder and Ogive Cylinder Geometric and Mass Characteristics; Memo to code 721.2 files at NASA-GSFC; 20 Sept. 1965.

3) Pitts, W. C., Nielsen, J. N.; Kaatari, G. E.; Lift and Center of Pressure of Wing-Body-Tail Combinations at Subsonic, Transonic, and Supersonic Speeds; NACA TR-1307; G. P. O., Washington, D. C.; 1953.

4) Miles, J. W.; Unsteady Supersonic Flow; A. R. D. C.; Baltimore; 1955: Section 10.4.

5) McNerney, J. D.; Aerobee 350 Windtunnel Test Analysis; Space General Corp.; El Monte, Calif.; January 1963.

Compilation of Equations

Nose

Either ogive or cone

$$(C_{N\alpha})_N = 2$$

ogive

$$\bar{x} = .466\,L$$

cone

$$\bar{x} = \tfrac{2}{3}L$$

Conical Shoulder or Conical Boattail

$$(C_{N\alpha})_{CS\,or\,CB} = \frac{8}{\pi d^2}\left(S_2 - S_1\right)$$

$$\bar{X}_{CS\,or\,CB} = \frac{L}{3}\left[1 + \frac{1 - d_1/d_2}{1 - \left(d_1/d_2\right)^2}\right]$$

from front of shoulder or Boattail

Fins

Four Fins

$$(C_{N\alpha})_F = \frac{16\left(s/d\right)^2}{1 + \sqrt{1 + \left(\frac{2l}{c_r + c_t}\right)^2}}$$

Three Fins

$$(C_{N\alpha})_F = \frac{\sout{13.85}\ 12\left(s/d\right)^2}{1 + \sqrt{1 + \left(\frac{2l}{c_r + c_t}\right)^2}}$$

See Addendum #2 for correction to 3 fin equation

$\therefore$ For Both

$$C_{N\alpha} = \frac{4N\left(\frac{s}{d}\right)^2}{1 + \sqrt{1 + \left(\frac{2l}{c_r + c_t}\right)^2}}$$

$$\bar{x}_{F\,\left(\substack{\text{from tip of}\\ \text{root chord}}\right)} = \frac{x_t}{3}\,\frac{(C_r+2C_t)}{(C_r+C_t)} + \frac{1}{6}\left[C_r+C_t - \frac{C_r C_t}{C_r+C_t}\right]$$

Interference

$$K_{T(B)} = 1 + \frac{r_t}{s+r_t}$$

$$(C_{N_\alpha})_{T(B)} = K_{T(B)}\,(C_{N_\alpha})_F$$

Total Values

$$C_{N_\alpha} = (C_{N_\alpha})_N + (C_{N_\alpha})_{CS} + (C_{N_\alpha})_{CB} + (C_{N_\alpha})_{T(B)}$$

$$\bar{X} = \frac{(C_{N_\alpha})_N\,\bar{x}_N + (C_{N_\alpha})_{CS}\,\bar{x}_{CS} + (C_{N_\alpha})_{CB}\,\bar{X}_{CB} + (C_{N_\alpha})_{T(B)}\,\bar{X}_{T(B)}}{C_{N_\alpha}}$$

REFERENCES

1) Shapiro, A. H.; The Dynamics and Thermodynamics of Compressible Fluid Flow, Vol. 1; Ronald; New York; 1953.

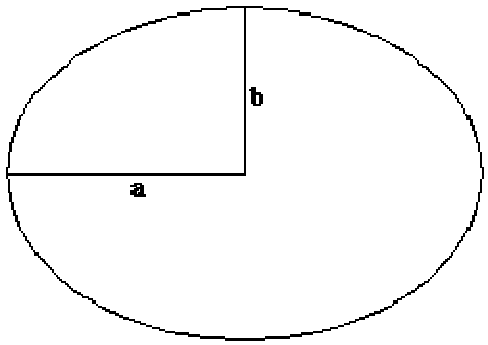
2) Mayo, E. E.; Cone Cylinder and Ogive Cylinder Geometric and Mass Characteristics; Memo to code 721.2 files at NASA-GSFC; 20 Sept. 1965.

3) Pitts, W. C., Nielsen, J. N.; Kaattari, G. E.; Lift and Center of Pressure of Wing-Body-Tail Combinations at Subsonic, Transonic, and Supersonic Speeds; NACA TR-1307; G. P. O., Washington, D. C.; 1953.

4) Miles, J. W.; Unsteady Supersonic Flow; A. R. D. C.; Baltimore; 1955: Section 12.4.

5) McInerney, J. D.; Aerobee 350 Windtunnel Test Analysis; Space General Corp.; El Monte, Calif.; January 1963.

The area of an ellipse is given by: $A = \Pi ab$ where a is the semi-major axis and b is the semi-minor axis of the ellipse.



For a single elliptical fin we want just half the area of the ellipse, so

$$A_f = \Pi ab/2$$

The semi-major axis is the span of the single fin, therefore:

$$a = S$$

However, the semi-minor axis is half the length of the root chord of the fin, therefore

$$b = Cr/2$$

Substituting the above two relations into the formula for the fin area we get:

$$A_f = \Pi SCr/4$$

Thus, the fin area equation you used was the area of two fins, not one. This dows not affect the numerator since the fin area cancels out. However, in the denominator, the fin area is squared; therefore, the constant of the second term within the radical is one quarter what it should be. Given rounding errors, 4 X 0,405 = 1.62 is essentially the value 1.623 in my elliptical fin equation.

Best Wishes,

Jim B.

me for the current contest year.

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...TIROS Section (Crystal Lake, Illinois) ...ces as part of the annual Independence ... static display of various rocket kits built by ... launch system, and two tracking scopes. ...stration launch held in the afternoon. It is ... people saw the demonstration and display.

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... the official Section News reporter for ...ed this past fall, please be sure and have the ... sent in to NARSN as soon as possible.

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...s would also like to thank the following ... news or sending in correspondence for this ...will all bear with us and keep sending in your

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...S appears each month as a regular feature ...CKETEER. Those sections wishing to have ... of their section activities printed in this ... such material to:

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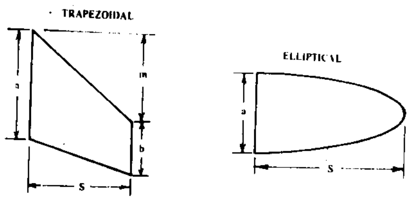
Elliptical Fin C.P. Equations

James S. Barrowman

In the last year or two, high performance model rockets have made use of elliptical planform fins. Elliptical fins are highly efficient because they produce a minimum of induced drag for a given amount of lift.

The fin normal force coefficient $(C_{N\alpha})_f$ and fin center of pressure $(\overline{X}_f)$ equations given in Centuri TR-33 "Calculating the Center of Pressure of a Model Rocket" are intended for a trapezoidal fin shape having the root and tip chords parallel. Trying to utilize the trapezoidal fin equations for elliptical fins is like trying to fit a round peg into a square hole.

Returning to the basic equations for generally shaped fins and applying them to the elliptical planform shape produces equations that are actually much simpler than those produced for trapezoidal fins. This occurs because the elliptical shape can be defined with half the number of dimensions required to define the trapezoidal shape.



Using the symbols from the Centuri report, the equations for elliptical fins are:

$$C_{N\alpha_f} = \frac{4n\left(\frac{s}{d}\right)^2}{1+\sqrt{1+1.623\left(\frac{s}{a}\right)^2}}$$

$$\overline{X}_f = X_f + 0.288\,a$$

The fin $(C_{N\alpha})_f$ chart given in Centuri TR-33 (Chart 4) can still be used to calculate the $(C_{N\alpha})_f$ of an elliptical fin. This is accomplished as follows:

Calculate $\frac{s}{d}$

Calculate $\frac{s}{a}$

Multiply $\frac{s}{a}$ by 0.638 to obtain an equivalent $\frac{1}{ab}$ values to Chart 4 in TR-33 to get the $(C_{N\alpha})_f$ for four fins.

As stated before, the elliptical fin equations were derived by applying the basic equation for a general shape to the elliptical shape. However, the elliptical fin $(C_{N\alpha})_f$ equation can be derived directly from the trapezoidal fin equation. How? Thats a brain teaser for the equation shufflers in the crowd.

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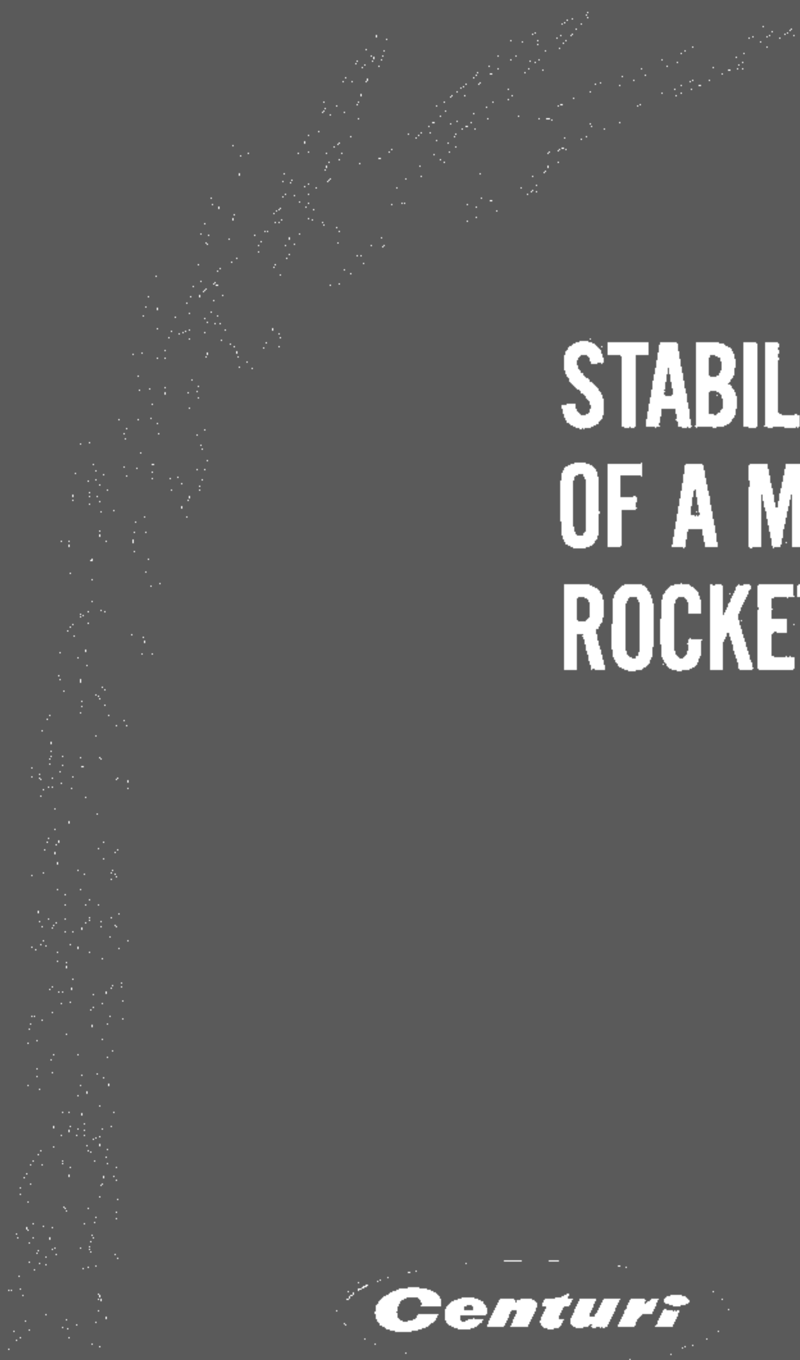

STABILITY OF A MODEL ROCKET IN FLIGHT

Centuri

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Centuri

TECHNICAL INFORMATION REPORT 30

STABILITY OF A MODEL ROCKET IN FLIGHT

BY JIM BARROWMAN

This report has been written to help you understand the scientific principles that affect the stability of your model rockets. It is not a "how to" manual on calculating stability. It has been written on the assumption that every model rocketeer wants to know "why" as well as "how to".

The best technique for accurately determining the stability of model rockets is given in CENTURI'S TIR # 33.

CENTURI ENGINEERING COMPANY
P.O. Box 1988
PHOENIX, ARIZONA 85001

ABOUT THE AUTHOR

JIM BARROWMAN is presently employed by NASA's Goddard Space Flight Center in Greenbelt, Maryland as an Aerospace Engineer in Fluid and Flight Dynamics. Jim was born in Toledo, Ohio 25 years ago and graduated from the University of Cincinnati in 1965 with a Bachelor of Science degree in Aerospace Engineering. Jim, together with his wife Judy and their two year old daughter Julie Ann, presently reside in Hyattsville, Maryland where he continues graduate level studies at nearby Catholic University of America.

He has been employed by NASA since 1961, and worked as a co-op student trainee during the first four years. Here he performed magnetometer data reduction for the Vanguard III Satellite, was a member of a Mars Atmospheric Entry Capsule design team, assisted in the thermal design of the IMP (Interplanetary Monitoring Probe) Spacecraft, performed dynamic motion studies of the Aerobee 150A Sounding Rocket, and wrote a computer program for his aerodynamic analysis of the Tomahawk, Nike-Tomahawk, and Black Brant IIIB Sounding Rockets.

Jim's interest in Model Rocketry dates back to 1964. He enjoys working with young people, and in addition to occasional lectures to Junior and Senior High School groups on aerospace careers, he has become the senior advisor for the NARHAMS Section of the National Association of Rocketry. The method Jim developed for calculating the exact center of pressure of a model rocket earned him a First Place Senior Research and Development Award at NARAM-8 in August, 1966. He is also an active NAR Trustee and has been appointed Chairman of the NAR Publications Committee and Contest Director for NARAM-10. Jim's obviously few leisure moments are spent sailing and experimenting with his favorite model rocket - - - the boost glider.

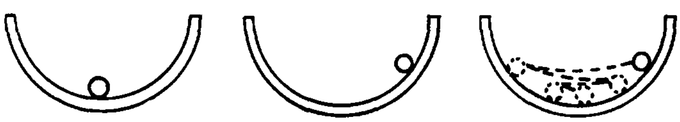
TABLE OF CONTENTS

1. INTRODUCTION

As a model rocketeer, you want your rockets to have the best and safest flights possible. In order to do this, you must know before you fly them that your rockets will fly straight up and not do loops around your head. In order to fly a straight and predictable trajectory, a model rocket must be stable. The basic rule for making a stable rocket is: A rocket will be stable only if its center of pressure is behind its center of gravity. This important rule won't help you if you don't understand what the words mean and how to use them. Also, you are probably wondering why such a rule is true. The answers to these questions are found by exploring the basic scientific principles that govern how model rockets move.

2. WHAT IS STABILITY?

As stated before -- to fly a straight and predictable trajectory, a model rocket must be stable. But, exactly what is stability? To get an idea what stability means, get a small rubber ball and a round bottom bowl. When you put the ball into the upright bowl, (see Figure 1a) it will sit at the very bottom of the bowl.



1a Neutral position | 1b Displaced (disturbed) position | 1c Stable motion

Figure 1

This is called its neutral position. Now, hold the ball on the side of the bowl (see Figure 1b). In this position the ball is said to be displaced or disturbed. You can see that if you hadn't moved it, the ball would have stayed at the bottom indefinitely. Any position in which a body will remain until it is disturbed is called a neutral position. If the ball were placed on a piece of corrugated metal, it would have many neutral positions, one in each trough.

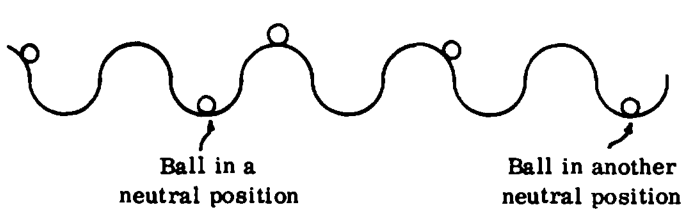
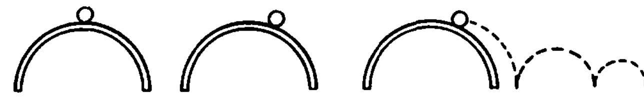


Figure 2

After displacing the ball, let it go and watch its motion. It will roll back and forth on the sides of the bowl until it stops at the bottom (see Figure 1c). It has returned to its original neutral position. The back and forth motion is called oscillation; and, because the ball returned to its neutral position, the oscillation is called positively stable or just stable. In general, any body that returns to its original neutral position after being disturbed is said to be stable.

Now turn the bowl upside-down on the table and very carefully set the ball at the top (see Figure 3a). Getting the ball to stay may be a difficult task; for if



3a Neutral position | 3b Displaced position | 3c Unstable motion

Figure 3

the ball is the slightest bit off the top (disturbed position, see Figure 3b) it will roll off the bowl and bounce away (see Figure 3c). If you succeed in getting the ball to set on top of the bowl, it will stay there in a neutral position. But, the smallest disturbance will cause the ball to again roll down and away. In general, then, any body that moves away from its original neutral position after being disturbed is said to be unstable.

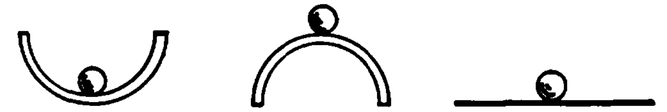
Finally, put the ball on a flat and level table (see Figure 4a). You see that it sits still no matter where on the table it was placed. That is, the ball is in a neutral position anywhere on the table. When it is

4a Neutral position | 4b Displaced position

Figure 4

displaced to another spot on the table (Figure 4b) it is again in a neutral position and does not move. It is neutrally stable. The associated general definition is: any body that is always in another neutral position after being disturbed from its original neutral position is said to be neutrally stable.

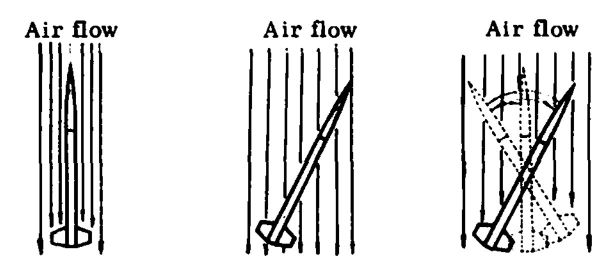
Thus, there are three types of stability; positive, negative and neutral. And, you can determine which kind of stability a body has just by watching how it moves.



Stable | Unstable | Neutral

Figure 5

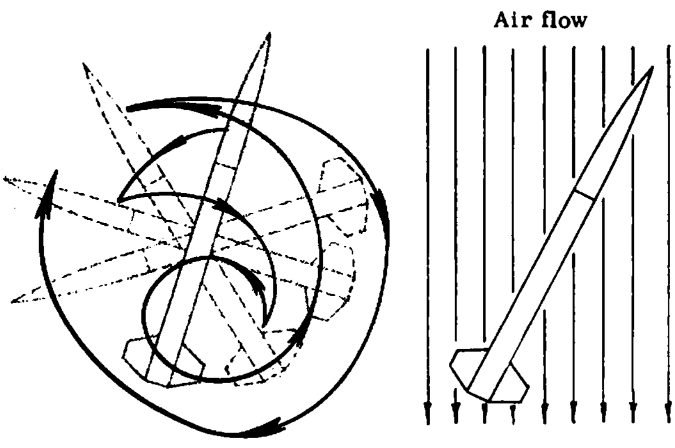
How does this apply to model rockets? Imagine a model rocket flying through still air. The air is passing smoothly over the model (see Figure 6a).



6a Neutral position | 6b Disturbed position | 6c Stable (oscillation)

Figure 6

As long as it isn't disturbed, the rocket will fly straight into the air flow. This is its neutral position. Now, if the rocket is hit, say by a wind gust from the side during flight, then it will fly at an angle to the air flow. This is its disturbed position. If the rocket then oscillates back to its neutral position (flying straight into the wind) it is stable. But, if it starts to fly at wider and wider angles to the air flow and eventually flips end-over-end in the air, it is unstable.



7a
Unstable
(flip-flops)

7b
Neutrally stable
(no motion)

Figure 7

If the rocket is neutrally stable, it will continue to fly at an angle to the air flow, no matter what the angle is.

Of course, what is shown in Figures 6 and 7 is what you would see if you were flying along with the rocket.

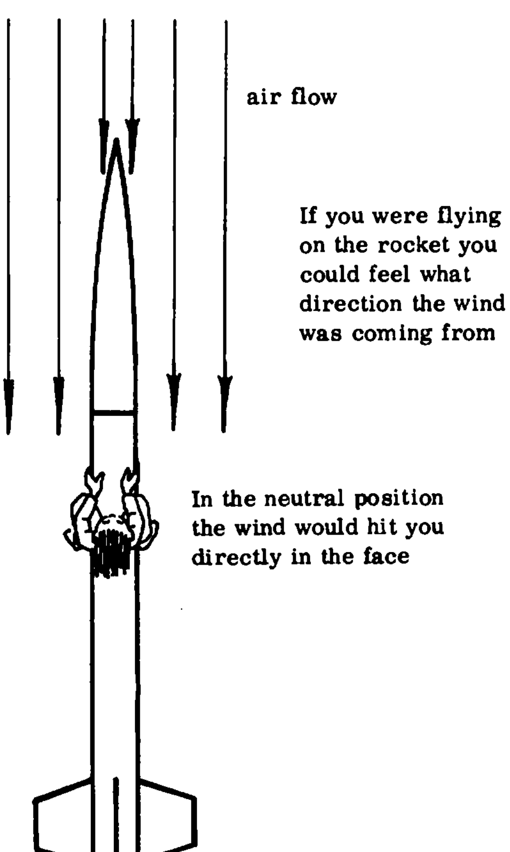


Figure 8

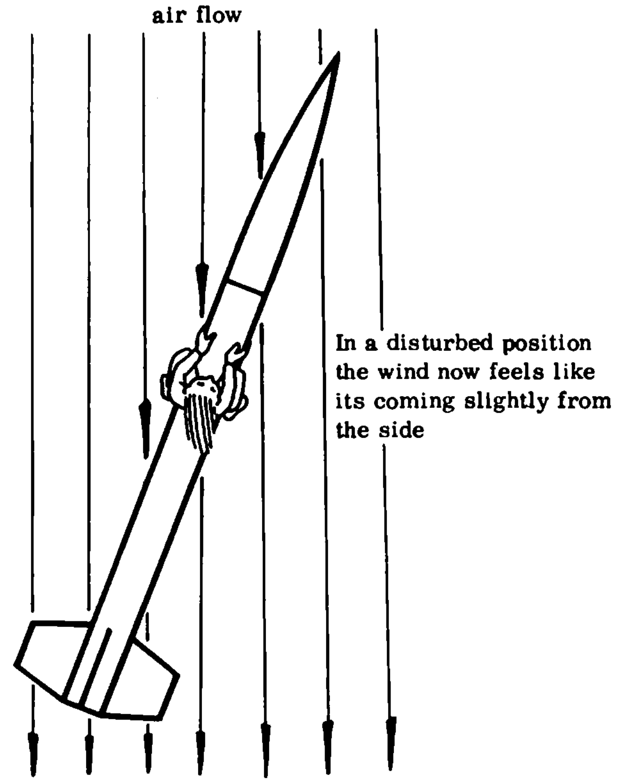
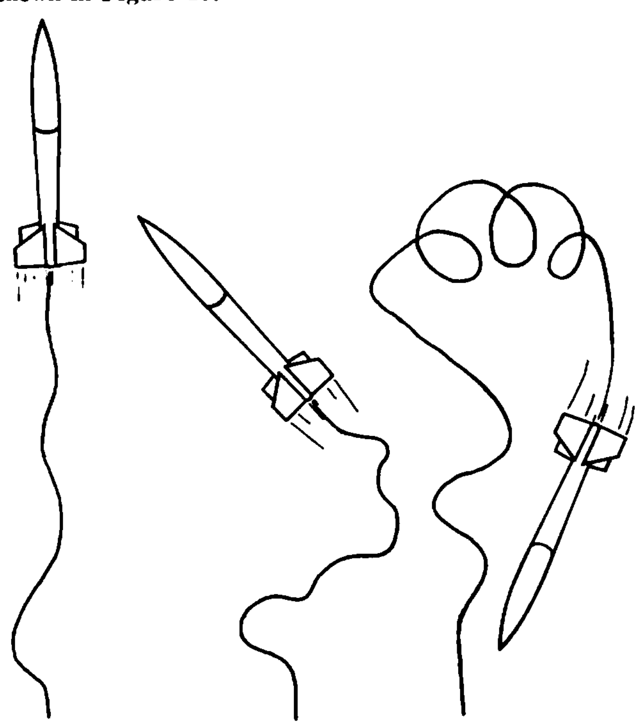


Figure 9

What you would really see while watching from the ground is shown in Figure 10.



10a
Stable
(oscillation)

10b
Neutrally
stable

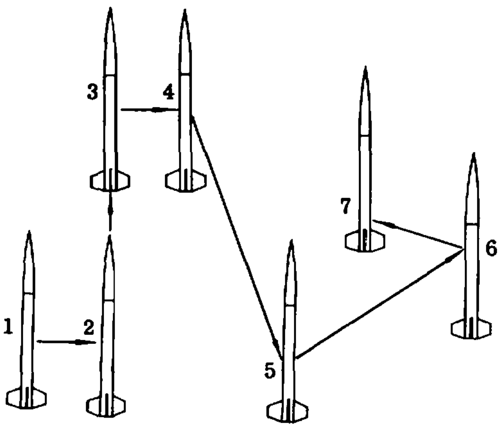
10c
Unstable
(flip-flops)

Figure 10

Stability in general, then, is a description of how a body (in this case a rocket) will behave while it is in motion. Simply by stating that a rocket is stable or unstable, you can describe a very complex pattern of motion.

3. ROCKET MOTIONS AND FORCES IN FLIGHT

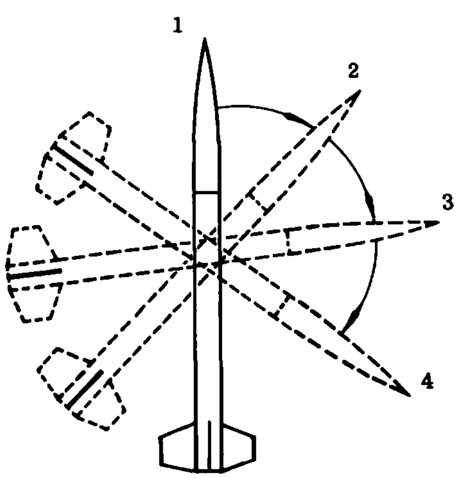
The motion of any body can mentally be separated into two different kinds of motion, translational and rotational. Figure 11 shows some examples of the translational motion of a rocket.



Translational motion

Figure 11

Notice that the rocket moves sideways, up, down, and across to different places but it always points in the same direction. The rotational motion of a rocket is shown in Figure 12. In this case, the rocket points in different directions while it stays in the same place.



Rotational motion

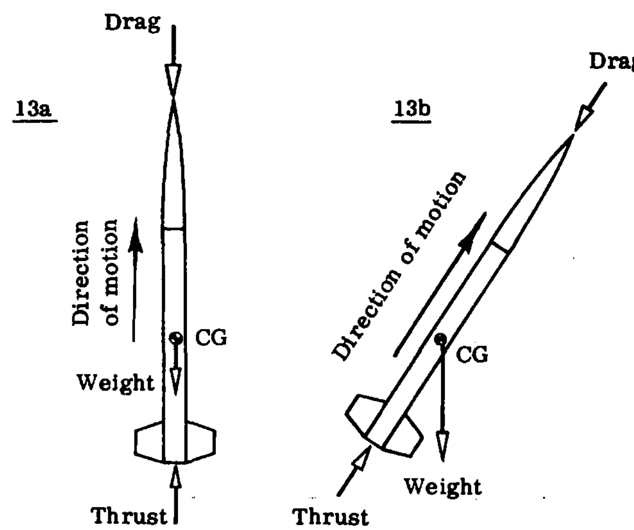
Figure 12

Rotation of any body is always about a straight line or axis. For example, a wheel rotates on an axle or a propeller rotates on a shaft.

When you ask how high a rocket goes or how far away it lands, you are interested in the rocket's translational motion. On the other hand, when you talk about a rocket's stability, you are concerned with its rotational motion. The real motion of a rocket, of course, is a simultaneous combination of its translational and rotational motions.

Any motion of a body is caused by the forces acting on it. The forces acting on a model rocket are its weight, the rocket engine's thrust; and the air pressure forces caused by the air flowing over the rocket. Just as the motions of the rocket can be mentally divided into two kinds, even though they occur simultaneously, so can the forces acting on it. They can be broken up so that there is a set of forces associated with the translation of the rocket and a separate set associated with its rotation.

The forces associated with translation are the rocket's weight; the engine's thrust; and the resistance of the air to the rocket's motion, called the aerodynamic drag. These forces are shown schematically in Figure 13. Notice that thrust is along the length of the rocket, drag is opposite the direction of motion, and weight always points down toward the ground.



Translational forces in vertical flight

Translational forces in slanted flight

Figure 13

Note also that the translational forces all act through the Center of Gravity (C.G.) of the rocket.

The forces associated with the rotational motion of the rocket are all those forces that do not act through the rocket's center of gravity. These forces are essentially the air pressure forces that act perpendicular to the rocket's centerline such as the lift on the fins and the nose. All these forces can be added together, and the total air pressure force acting perpendicular to the centerline can be considered to act at the center of pressure (C.P.). In Figure 14, this total force is represented by the letter N. The C.P. and C.G. locations are shown as they would be on a stable rocket using the standard symbols, ⊕ for C.G., and ⊙ for C.P.

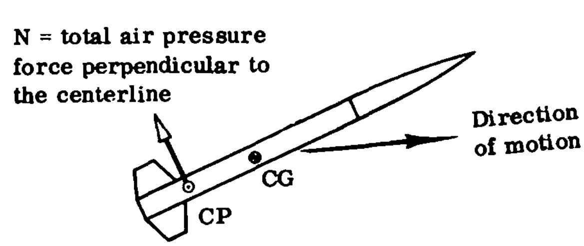


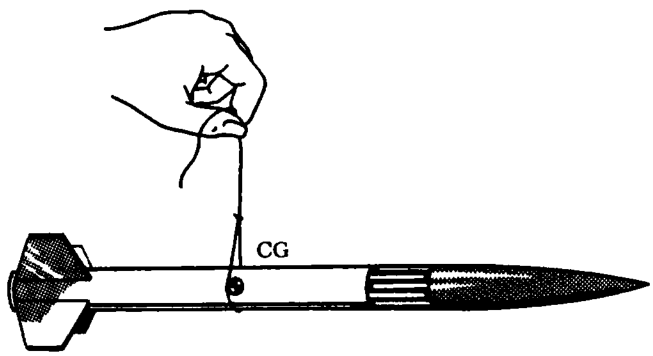
Figure 14

Notice that the rocket is flying in one direction while it is pointed in a different direction. It is in a disturbed position. The rotational forces on a rocket act only when it is in a disturbed position. The difficult thing about the rotational forces acting on a rocket is that while the forces affect the rotational motion of the rocket, the rotational motion, in turn, affects the forces and the C. P. location.

The division of a rocket's motion into two different types of motion leads directly to the separate consideration of the center of gravity (C.G.) and the center of pressure (C.P.). Each of these quantities is discussed in Sections 4 and 5. Their interrelation and its effect on model rocket stability are considered in Section 6.

4. THE CENTER OF GRAVITY

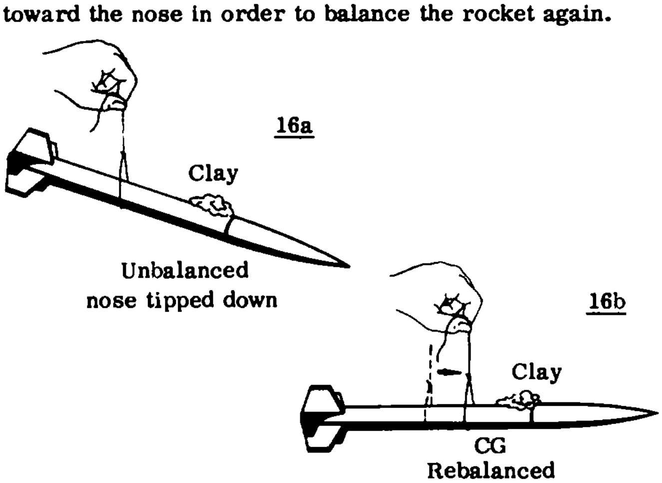
The center of gravity (C.G.) of a rocket is the point at which all the weight of the rocket seems to be concentrated. That is, there is as much weight distributed ahead of the rocket's C.G. as there is distributed behind it. Another name for the C.G. is the rocket's balance point. If you tie a string to the rocket at the C.G., the rocket will remain level, or balance.

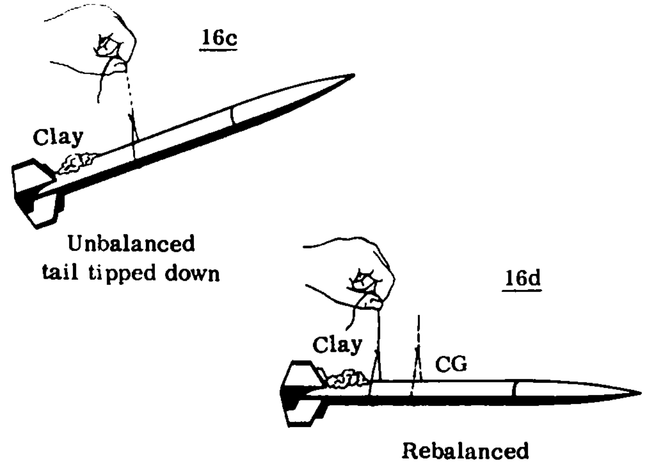


Balanced rocket

Figure 15

The force you feel on the string when you balance the rocket is the rocket's weight. This total weight is actually the sum of all the weights of the different pieces of material that make up the rocket. If you add more material to the rocket (like a lump of clay) it will weigh more. Also, if the additional material is attached, say to the nose, the nose will tip down and you will have to move the string toward the nose in order to balance the rocket again.



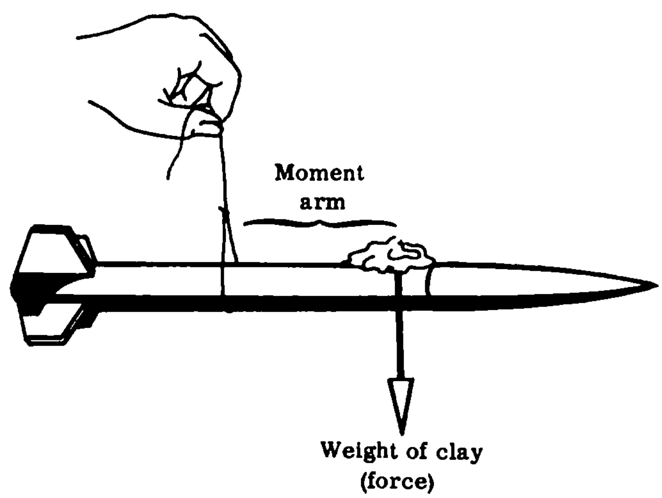


Addition of material

Figure 16

It is apparent that the weight of the material does more than add weight to the rocket. It also tends to rotate the rocket so that the end on which the material was added will tip down.

The tendency of any force, such as the weight of the clay, to rotate a body is called a moment. The size of a moment depends on the force itself as well as the distance between the force and the axis about which the body is rotating. The distance involved is called the moment arm. In this case, the force involved is the weight of the clay while the moment arm is the distance between the clay and the point where the string is tied.

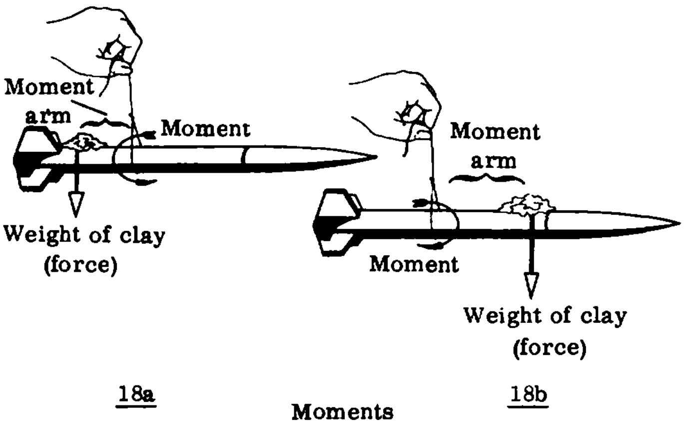


Force and moment arm

Figure 17

If you move the clay farther and farther from the string, the moment arm is increased and the rocket will tip more and more. Also, adding more clay to the same spot on the rocket increases the weight force and causes the rocket to tip more. In general, a moment is increased by either increasing the force or by increasing the moment arm. Mathematically, the size of a moment is the product of the force and the moment arm.

Moment = (Force) X (Moment Arm)



Moments

Figure 18

The curved arrow shown in both figures above is the standard symbol for a moment. Notice that they point in opposite directions in the two different pictures. You can see, then, that a moment not only has a size (force x moment arm), it also has a particular direction. The direction of a moment is the direction that it tends to rotate the body on which it acts.

Remember -- a moment is always about a specific line or axis. If you move the axis, you will change the moment arm and, therefore, the moment itself. When the string around the rocket is moved toward the clay, the rocket becomes level again. The moment caused by the clay weight has been reduced by reducing its moment arm. At the same time, a new moment acting in the opposite direction has been introduced. This new moment is simply the original rocket weight times its distance to the new balance point as shown below.

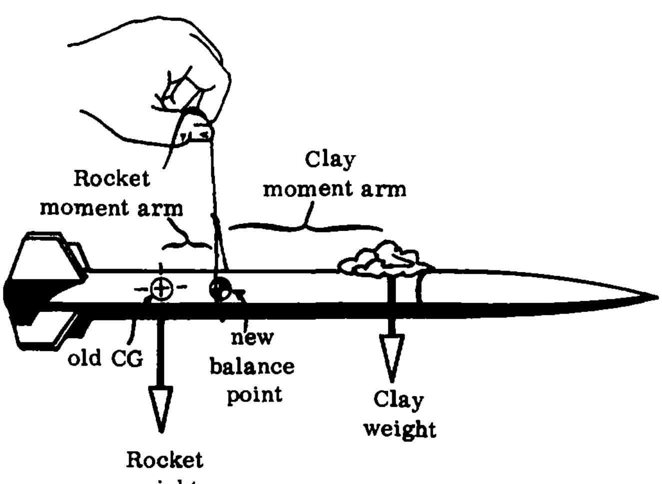


Figure 19

The new balance point is where the two moments are equal in size but acting in opposite directions.

$$\left(\text{Rocket Weight}\right) \times \left(\text{Rocket Moment Arm}\right) = \left(\text{Clay Weight}\right) \times \left(\text{Clay Moment Arm}\right)$$

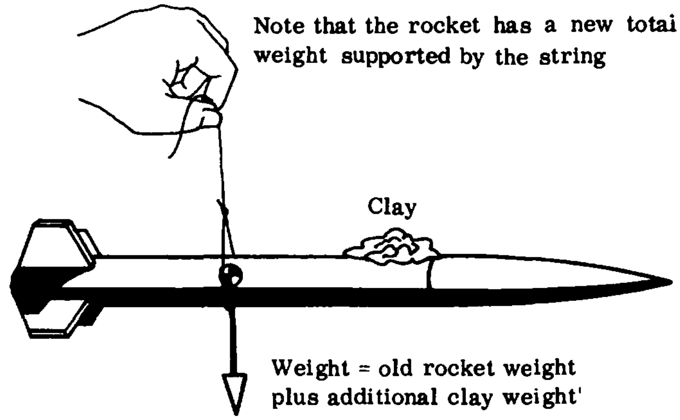


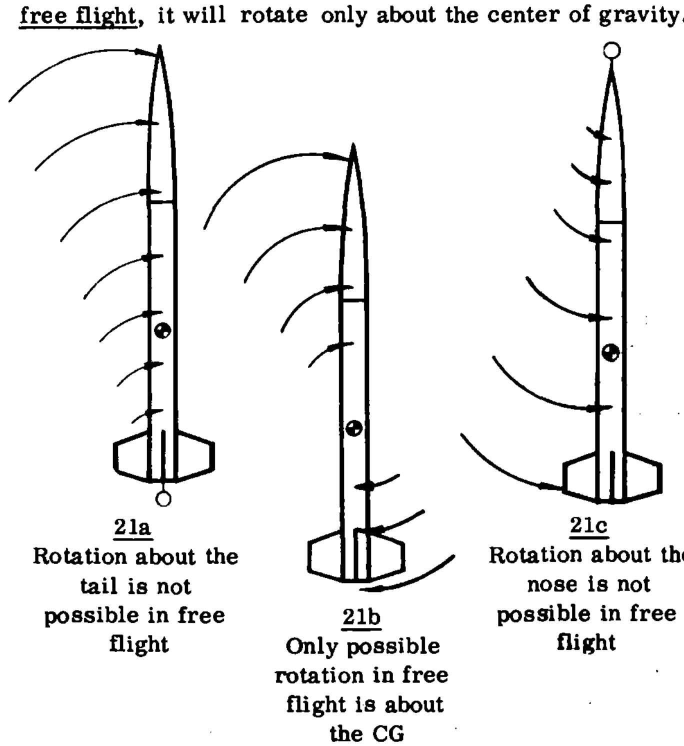
Figure 20

The new string position is the center of gravity of the rocket with clay attached to it (see Figure 16).

If a lump of clay added to the rocket has a moment associated with it, then each of the different pieces of material that make up the rocket must have a moment associated with it. At the beginning of this Section, it was stated that there is as much weight distributed ahead of the rocket's C.G. as there is distributed behind it. In terms of moments, this means the moment due to the weight of the material ahead of the C.G. equals the moment due to the weight of the material behind it. Thus, the rocket will remain level when you hang it by the C.G.

It is important for you to realize that the position of the center of gravity on a rocket (or any body) is associated with the distribution of the weight and not the weight itself. This same idea will be used in explaining the center of pressure.

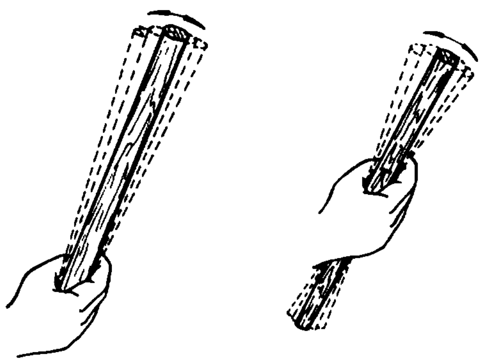
The center of gravity is important to stability not because the rocket balances there; but because when a rocket is in free flight, it will rotate only about the center of gravity.



Rotation about the center of gravity

Figure 21

Any body that is free to rotate in any way will naturally rotate about an axis through its center of gravity. You can force a body to rotate about a different axis by holding it, but then the body is no longer free to rotate naturally. A body will rotate about its C.G. simply because it is easier for it to rotate there than it is for it to rotate anywhere else. That is, it takes the least amount of effort to rotate a body about its center of gravity.* To see this for yourself, grip a long heavy stick at one end and swing it back and forth in front of you using only your wrist. Then grip it at its center of gravity and again swing it back and forth. You will find that it is noticeably easier to swing the stick about its C.G.



22a 22b

Figure 22

In review, there are two major facts that must be kept in mind about the C.G. when you are interested in model rocket stability. First, the position of the center of gravity of a rocket is determined by the distribution of the weight of the rocket. Second, when a rocket is flying free it will rotate only about its center of gravity.

5. THE CENTER OF PRESSURE

The center of pressure (C.P.) is similar to the center of gravity except that the forces involved are the air pressure forces acting on the rocket when it is flying. The C.P. can be defined in identically the same manner as the C.G. The center of pressure of a rocket is the point at which all the air pressure forces on the rocket seem to be concentrated. That is, there is as much air pressure force distributed ahead of the center of pressure as there is distributed behind it. In terms of moments, there is as much moment due to the air pressure force ahead of the center of pressure as there is behind it.

*As a matter of fact, everything that happens in the universe happens in such a way that the least amount of effort is required. This was mathematically proven by Sir W. R. Hamilton in the late Nineteenth Century. This basic physical principle is called Hamilton's Principle of Least Action. If you study advanced mechanics or physics in college you will learn about Hamilton's Principle and how it can be proven.

In the figure below, the size of the air pressure forces that are distributed over the length of the rocket and on the fins are represented by the length of the arrows along the top of the rocket.

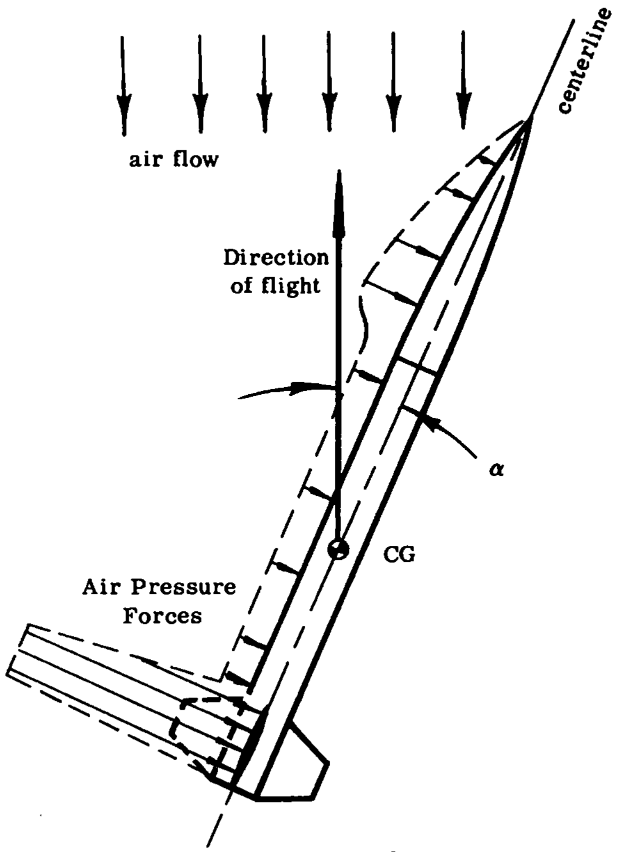


Figure 23

As you can see, the rocket is in a disturbed position (exaggerated in the drawing). That is, it is at a slight angle to the direction it is flying. As a result, it is at an angle to the direction of the air flow over it. This is called the angle-of-attack and is represented by the Greek letter Alpha, α. Notice that the air pressure forces pictured above are all perpendicular to the rocket centerline. These are called the normal (mathematical term meaning perpendicular) forces acting on the rocket. Simultaneously, there are also axial air pressure forces on the rocket. Although the axial forces are important in calculating the altitude performance of the rocket, they do not influence its center of pressure.

The distribution of normal forces shown above represents how the forces actually act on a typical model rocket flying at an angle-of-attack. However, as was mentioned in Section 3 on rocket motions and forces, these distributed normal forces can be added together and their combined effect on the model can be reproduced by a total normal force called N. Just as the total weight acts at the C.G., the total normal force acts at the center of pressure or C.P.

Figure 24 is equivalent to Figure 23 except that the distributed normal forces have been replaced by N acting at the C.P.

Just as the center of gravity location depends on the rocket's weight distribution, the location of the center of pressure depends strongly on the way the air pressure forces are distributed over the rocket.

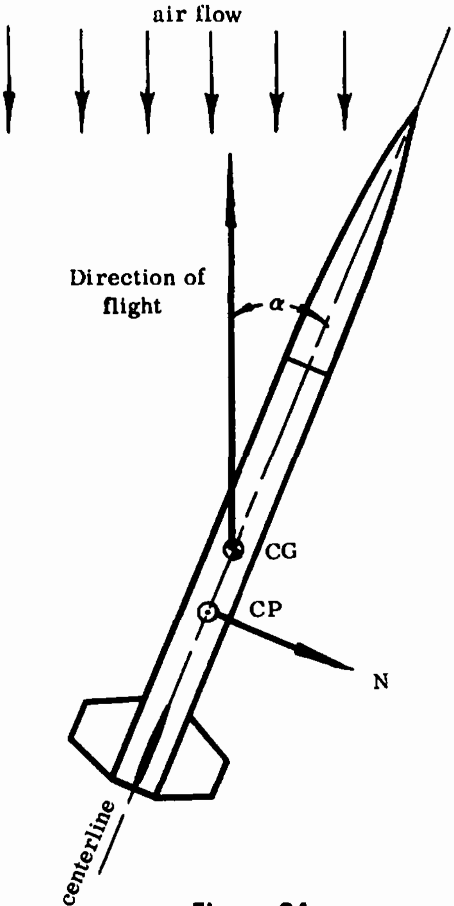
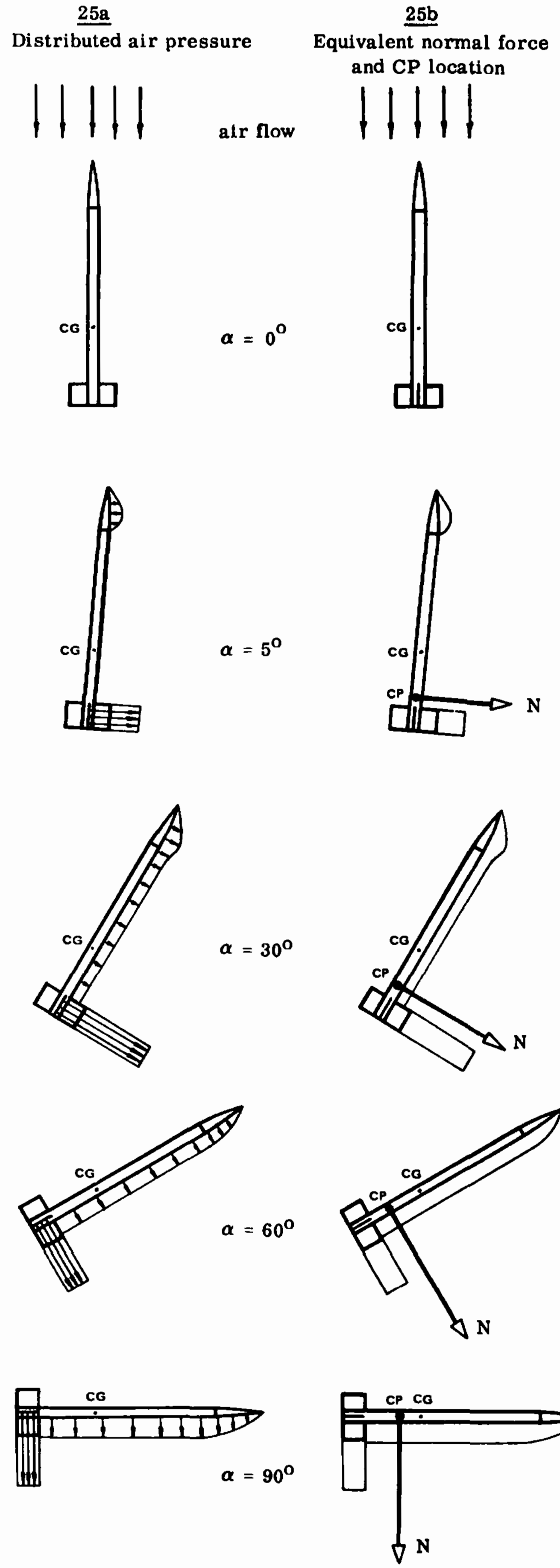


Figure 24

The angle-of-attack at which a rocket flies has a strong effect on the size and shape of the normal force distribution on a rocket. The left hand side of Figure 25 shows how the normal force distribution on a typical model rocket looks when it is flying at different angles-of-attack. The right hand side shows the equivalent normal force and C.P. location for each α . The equivalent total normal force gets larger as the angle-of-attack increases. But, more important, the distribution of the normal forces changes a great deal as the angle-of-attack increases. As the distribution changes, the center of pressure moves. As shown on the right hand side of Figure 25, the location of the center of pressure moves forward as the angle-of-attack increases.

In terms of moments, the increase in angle-of-attack causes the normal forces to build up all along the rocket. However, the moments of the normal forces near the nose increase faster than the moments of the normal forces near the tail. This means that there is no longer as much moment due to air pressure force behind the C.P. as there is ahead of it. Just as the C.G. has to move toward any added weight in order to decrease its moment arm and rebalance the rocket; the C.P. moves toward the nose so that the moments ahead of C.P. and behind it will again be the same.

The fact that the C.P. moves forward as the angle-of-attack increases is very important since it can affect a rocket's stability.

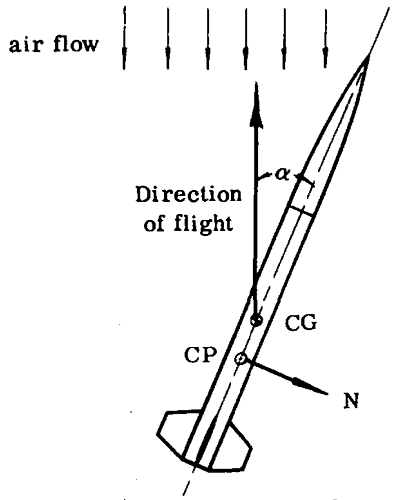


Variation of CP location with angle-of-attack
Figure 25

6. ELEMENTS OF MODEL ROCKET STABILITY

Remember from Section 4 that a rocket in free flight will rotate only about its center of gravity. Remember, also, that the only thing that can cause a rocket to rotate is a moment. The two things that determine a moment are a force and a moment arm.

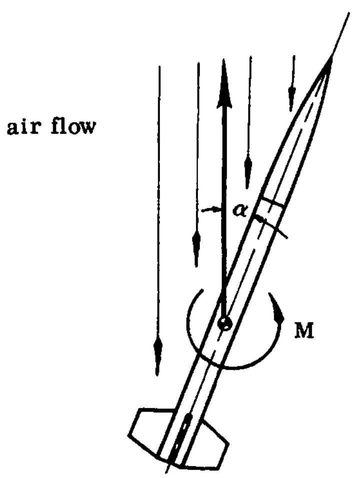
As shown in Section 5, a rocket flying at an angle-of-attack (disturbed position) has its total normal air pressure force acting at its center of pressure.



Normal force and CP
(stable condition)

Figure 26

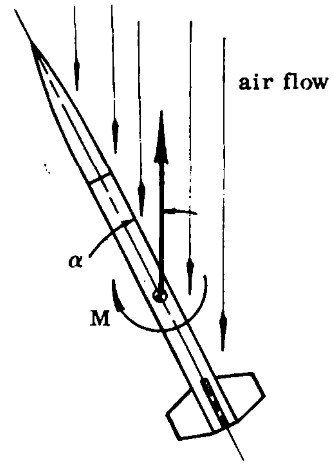
The normal force, N, and the distance between the center of pressure and the center of gravity (moment arm) combine to form a moment, M, about the center of gravity.



Aerodynamic moment
(stable condition)

Figure 27

Since the distance between the C.G. and C.P. is the moment arm associated with the normal air pressure force it is quite important and has been given the special name static margin. Since the moment, M, is associated with the air pressure forces, it is called aerodynamic moment. The aerodynamic moment tends to rotate the rocket about the center of gravity. In the figure shown above, the rocket will rotate back toward the direction of motion decreasing the angle-of-attack. Since the rocket is rotating as it reaches zero angle-of-attack, it will swing past its direction of motion and will again be at an angle-of-attack.



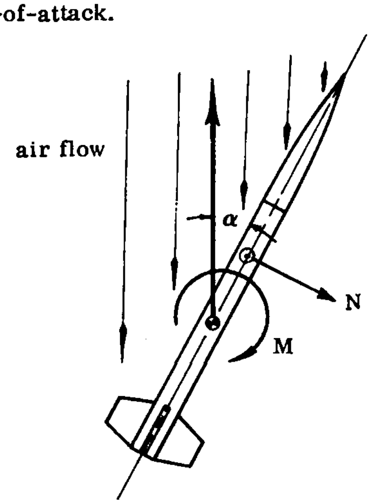
Stable oscillation

Figure 28

But, the normal force and resulting aerodynamic moment at the new α tend to again rotate the rocket toward the direction of motion. This process is repeated as the rocket swings back and forth less and less until it finally stops swinging and flys pointing in the direction of motion (zero angle-of-attack).

By now you may have recognized that a rocket at zero angle-of-attack is in its neutral position and that the process described above is a stable oscillation such as the one described in Section 2. Notice in the figures shown above that the rocket's C.P. is behind its C.G. This is exactly the requirement for stability that was stated at the beginning of this report -- A rocket will be stable only if its center of pressure is behind its center of gravity.

But, what will happen when the C.P. is ahead of the C.G.? The figure below shows this situation on a rocket that is at an angle-of-attack.

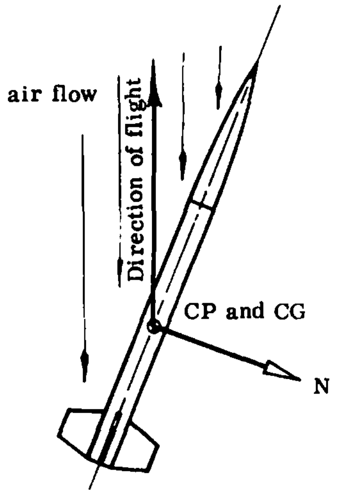


Aerodynamic Moment
(unstable condition)

Figure 29

In this case, the aerodynamic moment tends to rotate the rocket away from its direction of motion (neutral position). From the associated definition in Section 1, it is obvious that a rocket that has its C.P. ahead of its C.G. is unstable and will flip-flop all over the sky.

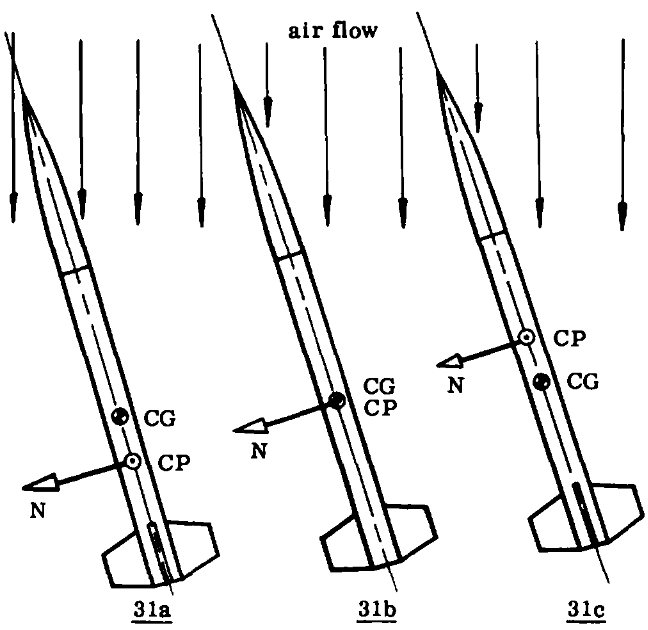
Having looked back at Section 1, you may wonder what C.P.-C.G. relationship corresponds to neutral stability. To be neutrally stable, a rocket must remain at any angle-of-attack at which it is flying. In order to remain at any given angle-of-attack a rocket must have no tendency to rotate. Thus, there can be no moment associated with the normal air pressure force acting on the rocket. For this to be true, the static margin must be zero; or, the C.P. must be at the same location as the C.G.



Neutral stability (M = 0)

Figure 30

In review, the three types of model rocket stability and the associated C.P.-C.G. relationships are shown below. Using arrows to represent the normal force obviously helps in visualizing the rotation about the C.G.



31a Stable — 31b Neutral — 31c Unstable

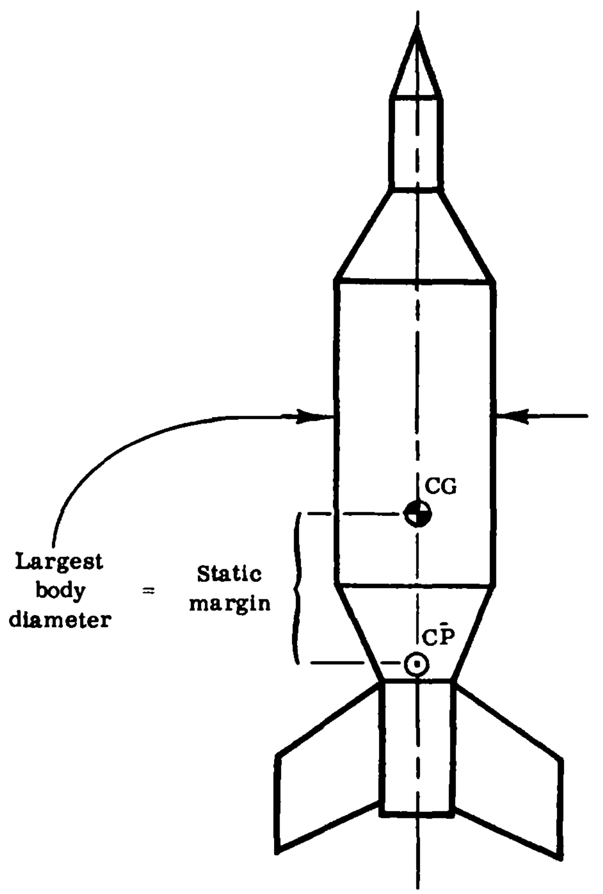
Types of stability

Figure 31

Knowing that your rocket is stable is not enough. You must also know how much stability it has. That is, how quickly does the rocket oscillate back to its neutral position after it has been disturbed. Since it is the moment arm for the aerodynamic moment, the static margin has the greatest effect on a rocket's stability. Thus, for a stable rocket, the larger its static margin, the more stable it will be.

The importance of the effect of angle-of-attack on the position of the C.P. now becomes apparent. As a rocket's angle-of-attack increases, the C.P. moves forward, the static margin decreases, and the rocket becomes less stable. It's possible that the C.P. might even move forward of the C.G. causing the rocket to become unstable. Obviously, you want your rockets to have a static margin that is large enough to insure that it will be stable and fly at small angles-of-attack.

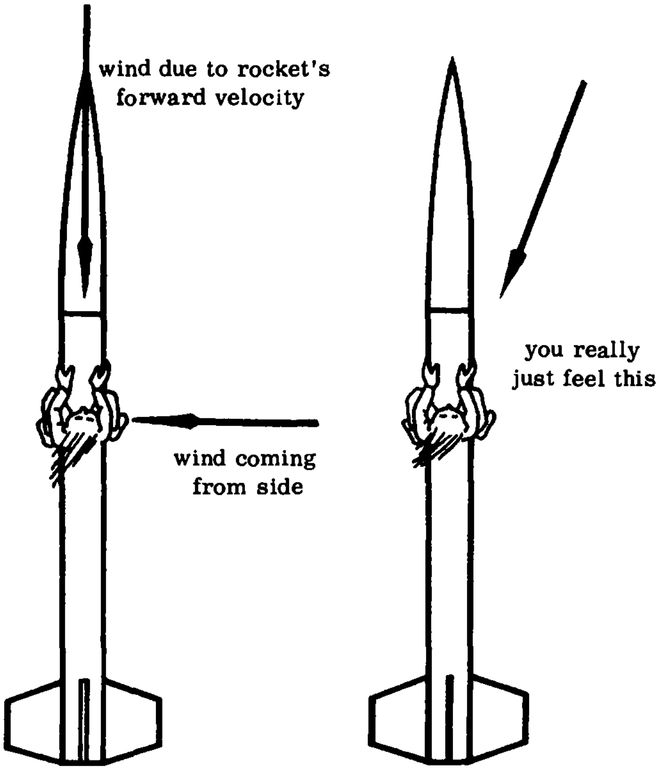
But, if your rocket is too stable, it will weathercock too easily. A good rule of thumb is to have the static margin equal to the largest diameter of the rocket.



Ideal static margin

Figure 32

The reason a rocket weathercocks or consistently arcs over on a windy day is that the rocket is at an angle-of-attack to the wind even as it leaves the launch rod. If you were riding along with the rocket it would seem as if the wind was off at an angle to your face even though you know you were leaving the rod going exactly straight up.

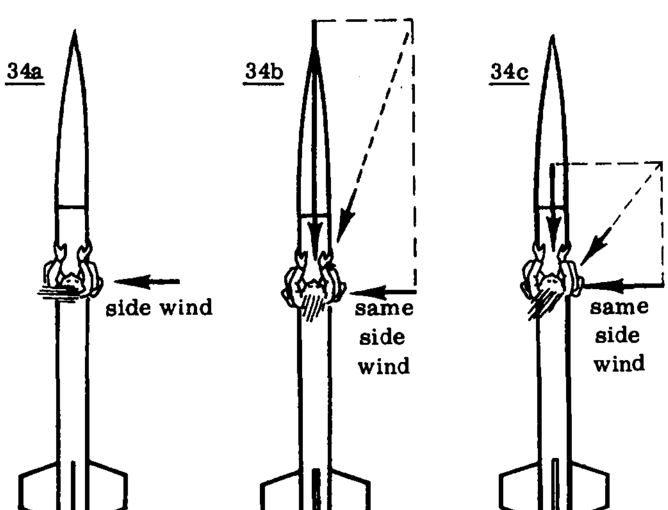


33a 33b

Weathercocking

Figure 33

The wind is not just from the side as you felt it just prior to lift-off. The total wind is now the wind due to the rocket's velocity forward plus that of the side wind. The effect of the side wind on this angle is more pronounced if you were riding a heavier rocket which leaves the rod at a much slower velocity.

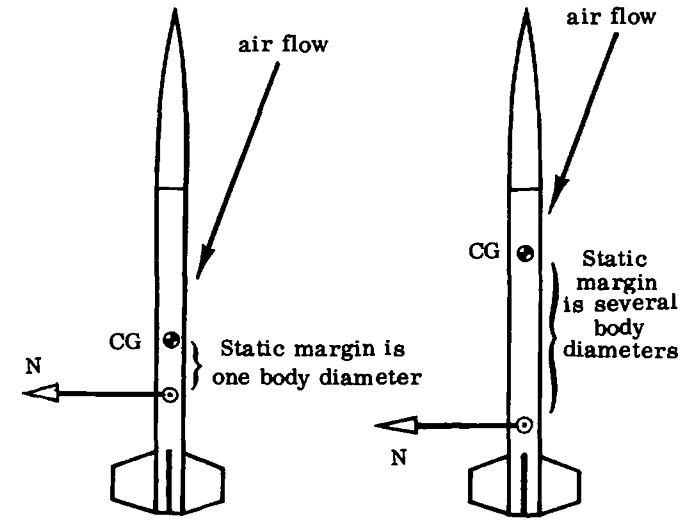


before lift off | High performance rocket as it leaves the launch rod | Slow heavy rocket as it leaves the launch rod

Figure 34

Now that the rocket's motion is not restrained to straight up any more by the launch rod it can rotate freely about its natural free flight pivot point, which is the C.G. Since the wind is hitting the rocket at an angle, a normal force is produced. If the rocket is stable (C. P. behind C.G.), the resulting moment tends to rotate the rocket directly head on into the wind, thereby gradually reducing the angle-of-attack of the wind to zero.

Obviously, the larger the moment due to the normal force acting at a distance from the C.G., the quicker the rocket responds and rotates all the way to zero angle.

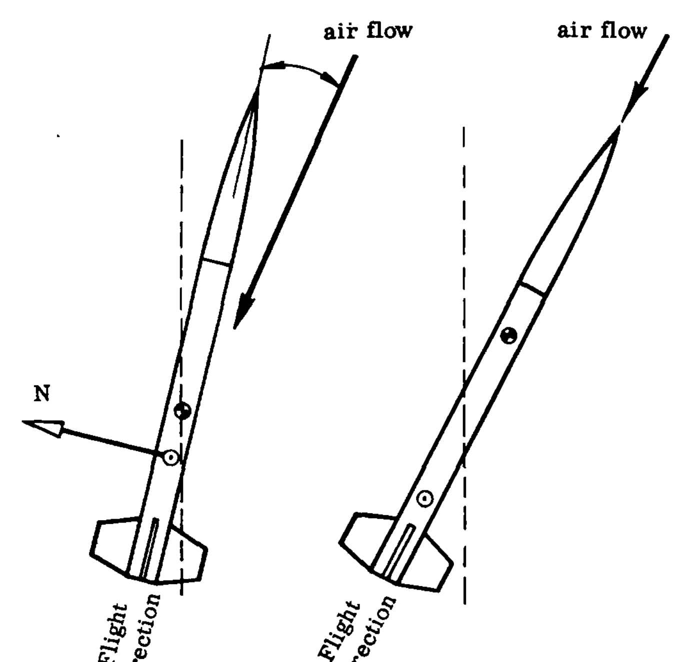


35a Stable | 35b More stable

Figure 35

The above illustration shows that the moment about the C.G. will indeed be larger for the more stable rocket, because even though the forces are identical, the moment arms are different.

One second after leaving the rod the more stable rocket may already be aligned to zero angle-of-attack while the rocket with a static margin of stability of only one body diameter may still be completing its correction because of its smaller restoring moment.



36a 36b

Figure 36

from the ground we would see

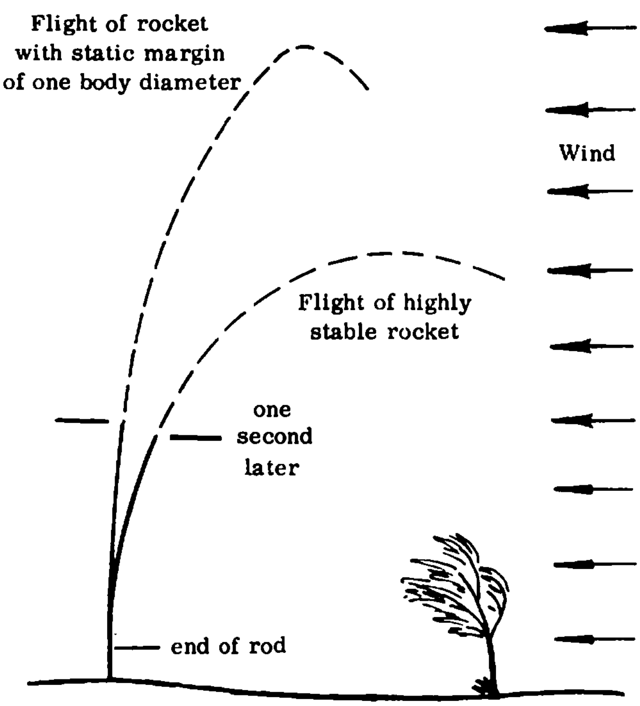


Figure 37

Once the causes for weathercocking are understood, we can immediately see ways to reduce the effect:

a) Use a longer than usual launch rod so that the rocket has more time to build up forward speed. The larger the speed, the less the angle for a given crosswind.

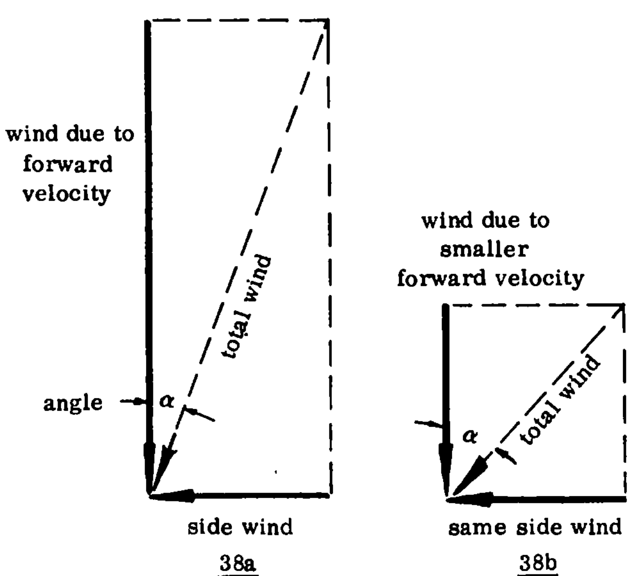


Figure 38

b) Use rockets with static margins not much greater than one largest body diameter. More stability will mean more weathercocking tendency.

c) Use the highest average thrust motor possible so that the velocity when the rocket leaves the restraint of the rod is as large as possible.

7. DETERMINING STABILITY

There are a number of dependable methods that you can use to determine the stability of your rockets before you fly them. They range from purely experimental to purely theoretical.

SWING TEST

The best experimental method is to swing the fully prepared rocket (parachute and engine inserted) on a string attached to the rocket at its center of gravity. First, balance test the rocket to find its C.G. Then secure the string firmly at the C.G. with a piece of tape or straight pin. Check to see if the area is clear so you won't hit anyone or smash the rocket when you swing it. Be careful to start the rocket so that it is pointing in the direction that it is moving. Remember that if it has a large angle-of-attack, it may be unstable at that angle even though it is actually stable at smaller angles-of-attack. If the rocket tends to stay pointed in the direction it is moving as you swing it, then it is stable. Several tries may be necessary before you can start the rocket swinging smoothly; but if the rocket simply will not stay pointed in the direction it is moving, then it probably doesn't have adequate stability.

LOCATING THE CENTER OF GRAVITY

Without actually swing-testing the rocket, you can separately determine the rocket's center of gravity and center of pressure. Then if the C.P. is properly behind the C.G. the rocket will be stable. The C.G. location can be found by balance testing it, as shown in Figure 15. However, if you are designing a rocket and don't have it completely built, then a theoretical technique must be used. Such a technique for determining a rocket's C.G. location is given in Centuri's Technical Report TIR-33.

CARDBOARD CUT-OUT METHOD FOR LOCATING THE CENTER OF PRESSURE

There are two major methods available for determining the location of your rocket's center of pressure location. The best technique for beginners is to find the rocket's Center of Lateral Area (C.L.A.). This is the position the C.P. would have if the rocket were flying at an unrealistic angle-of-attack of 90°. Thus, it is the forward most position that the C.P. can have. To find the C.L.A. of a rocket, draw an outline of the rocket including the fins on a sheet of stiff cardboard. Cut the outline out and balance it on a ruler or with a pin as shown below:

39a

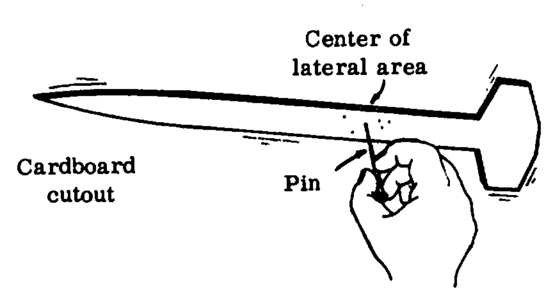


Figure 39

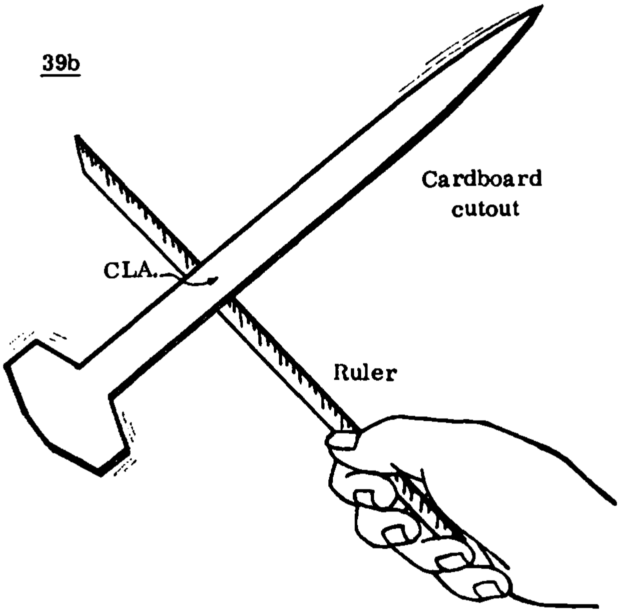


Figure 39

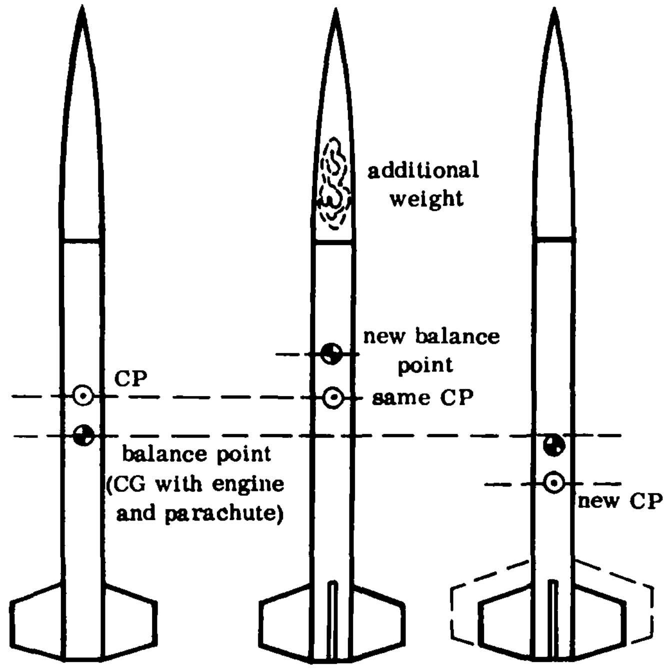
The point where the cutout balances is the rocket's C. L. A. If the cardboard is not stiff enough it will just sag and won't give the proper balance point. If the C. L. A. location of the rocket is behind its C.G. location, then the rocket will definitely be stable. A static margin of one half ($\frac{1}{2}$) the rocket's largest body diameter is quite adequate when you are using the C. L. A. cutout method.

Of course, a stable model rocket will never fly at an angle-of-attack of 90°. Actually, the static margin of a stable rocket will be larger than indicated by the center of lateral area location. Thus, if you use the C. L. A. cutout method, your rockets will tend to be overly stable and weathercock very easily. To prevent this, you can theoretically calculate your rocket's true C. P. location when it is flying at angles-of-attack near zero. A method for accurately determining a model rocket's true C. P. location is given in Centuri's TIR #33.

When working with the true C. P. location you must always be careful to provide an adequate static margin. Remember that one maximum body diameter is the smallest safe static margin.

8. OPTIMIZING STABILITY

Every rocketeer is concerned with the question of how high will his rocket go. The more sophisticated modeler realizes that stability requirements can and will affect altitude performance. If a rocket checks out as unstable, one has two alternatives to correct this. The first is to add weight to the nose to bring the C.G. adequately ahead of the C.P. As you know, though, the heavier a rocket is the lower the altitude it can attain. The second alternative is to increase the fin area in an attempt to shift the C.P. safely aft of the C.G. While this technique adds less overall weight to the rocket it adds aerodynamic drag thereby also reducing potential altitude. Besides, it has the disadvantage of requiring reconstruction in the case of a completed model.



40a
Unstable configuration

40b
Add weight
to nose

40c
Increase fin area
(note that CG shifts
slightly to the rear)

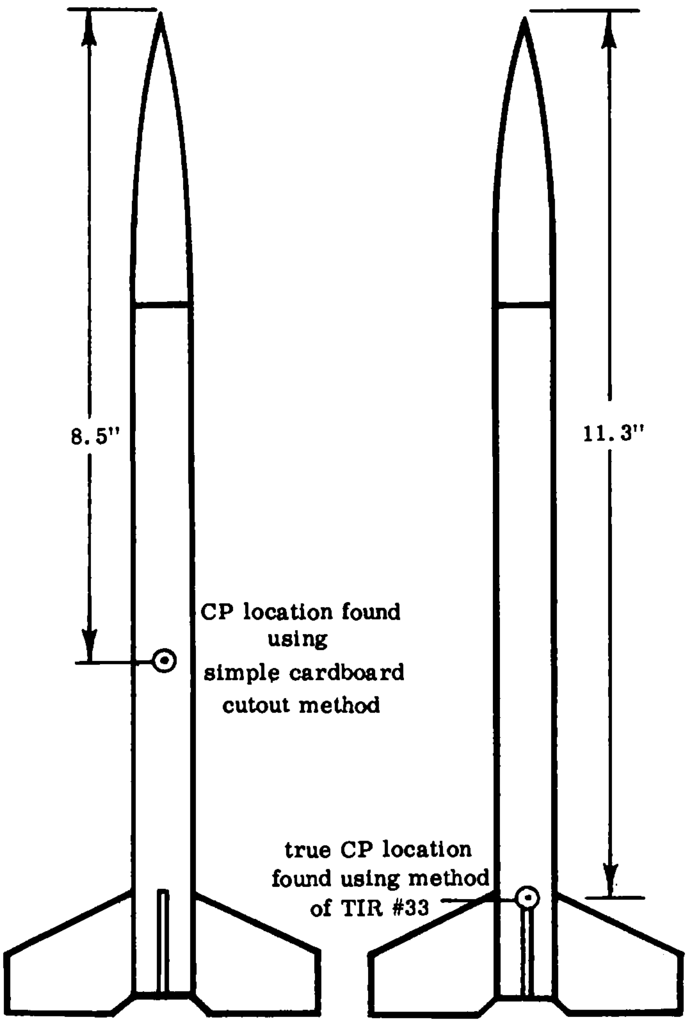
Techniques for making an unstable rocket stable

Figure 40

Using the criteria that the static margin (or distance the C.G. lies ahead of the C. P.) must be at least one maximum body diameter, it is easy to see that the C.P. location strictly dictates the C.G. location. A rocket with a larger static margin is obviously carrying more weight and/or has more fin area than is absolutely necessary. Don't forget that the weathercocking tendency and consequent reduction in peak altitude is also more prevalent for the excessively stable rocket.

If you are interested in comparing the exact effects that weight savings and aerodynamic drag improvements have on peak height, it is recommended that you obtain Centuri's TIR #100 which contains altitude performance graphs. No mathematics at all is required to find heights using these graphs, but the answers are restricted to idealized perfectly straight-up flights.

The competition oriented rocketeer should be especially aware of the influence of the method used in determining the center of pressure (C. P.) location. The simple cardboard cutout method gives a C. P. location for a rocket flying sideways, which may be one half to as much as four maximum body diameters ahead of the C. P. found using the method that gives the true C. P. for a rocket flying at zero angle-of-attack.



CP locations for Centuri's four-finned JAVELIN

Figure 41

Admittedly, there is more effort involved in finding the C.P. using TIR #33, but from the above illustration it can be seen that the choice of method used to find C.P. results in a considerable difference for the definition of a safe C.G. location. In order to check out as stable, a rocket whose C.P. has been found using the somewhat crude cardboard cutout method will require a more forward C.G. location than if the C.P. was found for the aligned flight condition. Keeping in mind that the true C.P. is the only one of real concern we can be quite sure that the rocket whose static margin is based on the true C.P. will be the better performer, as less weight penalty exists.

It should be mentioned at this time that if the C.G. was naturally ahead of the true C.P. by more than one body diameter to start with, we would not improve performance by adding weight to the tail to shift the C.G. closer to the optimum flight performance location. The rocket would be fine to fly as is, but in the interest of better performance one could consider removing weight near the nose (perhaps hollowing the nose cone itself) and removing fin area to improve aerodynamic efficiency.

At present, no one has established a simple fudge factor to convert the C.P. found using the cardboard cutout method to the true C.P. location. The difference between the two methods, using the number of maximum body diameters as a measurement, varies with each and every rocket. Safety conscious model rocketeers should not guess at the true C.P. location, especially when the tool for actually finding it is available. As previously stated, you can safely use a static margin of one half of a maximum body diameter with the cardboard cutout method, while the true C.P. requires one maximum body diameter for its static margin.

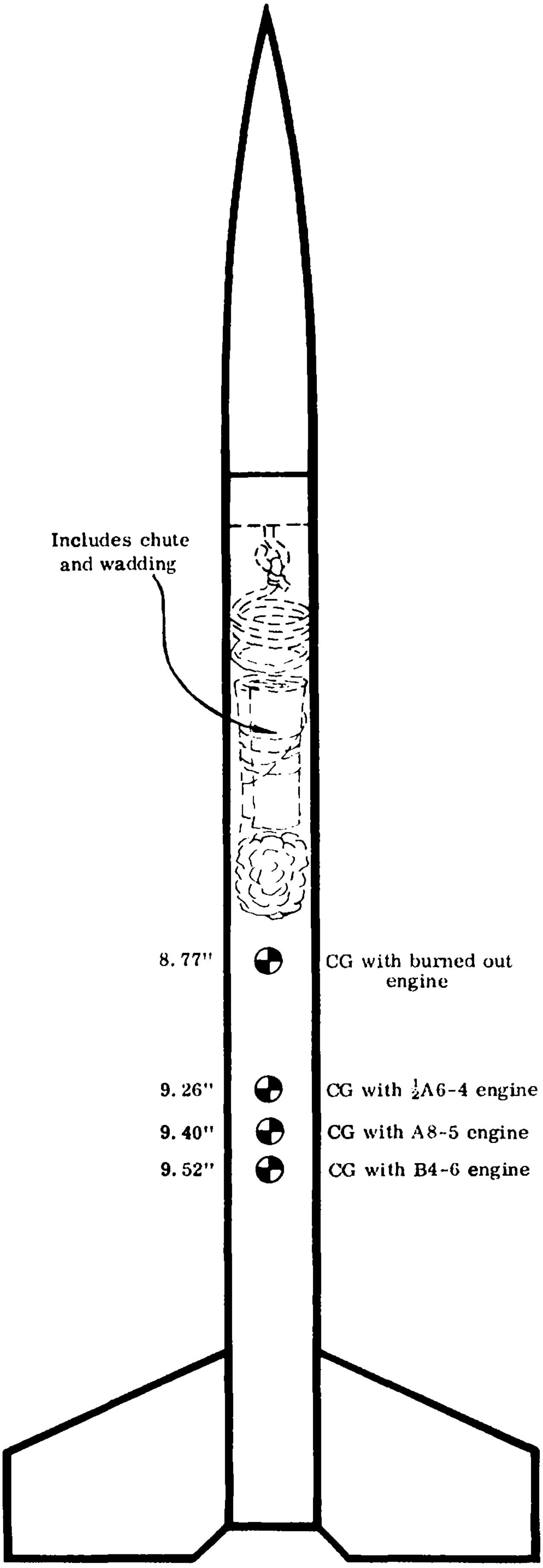
9. SAFETY

Occasionally, model rocketeers fly their new custom rocket designs without bothering to check that the static margin is adequate. They think that they have enough experience to judge whether or not a design is stable just by looking at it and are quite surprised when the first flight sometimes proves to be unstable. The rocket zig-zags and does flip-flops all over the sky immediately after leaving the launch rod and as a result the thrust is just wasted in random motion.

This approach to model rocket design is not very scientific and obviously gives no consideration to optimizing performance. Also, the time spent in repairing the damaged model after it falls back to the ground usually more than offsets the time required to do a simple stability check. A serious scale modeler would never think of risking his tremendous investment in construction time in such an unscientific manner. The proper approach with any new design is to first find the rocket's C.G. and C.P. so that stability and adequate static margin can be verified; then test fly the rocket with the lightest acceptable engine on a calm day.

You may ask "Why use the lightest acceptable engine?" Remember the propellant -- it is all at the back end and it is burning away. It is like a lump of clay on the back which is being removed a little bit at a time. If the rocket were balanced on a string, the balance point would have to be shifted forward slightly after each particle of clay was removed. The balance point on the string is the C.G. and the rocket will rotate only about that point during free flight. During an actual flight, the consumption (burning) of the propellant similarly causes a continuous shift in the rocket's balance point (or center of gravity, or pivot point, or only point where rotation occurs -- whatever makes it easiest for you to understand).

In other words, the C.G. location is not fixed. The following illustration shows the different initial balance points of a Javelin with various motors and also shows the final C.G. location after all the propellant and smoke delay is used up.



Centuri JAVELIN variations in center of gravity

Figure 42

If you never previously considered the idea of a C.G. shift during flight, you were probably intrigued to see that it actually shifts that much. Thus, one of the consequences of using a lighter engine in the first flight is that you will have a little more static margin of safety. It can also be concluded that a rocket which is stable initially will become more stable as the flight progresses because the C.G. is moving further and further ahead of the C.P.

One additional thought to consider arises occasionally from a rather special set of circumstances. Referring to the previous illustration of the Javelin -- what could be said about an initial flight with a B4-6 engine if the true C.P. location was not at the previously shown location of 11.3 inches but instead at an assumed 9.40 inches from the nose.

Initially, the C.G. is behind the C.P. and the rocket is unstable. If the rocket is going to fly unstable it will usually do so immediately upon lift-off. All the while the rocket is tumbling and flip-flopping around the sky the C.G. is shifting forward. Soon the C.P. and C.G. match at 9.40 inches -- meaning the rocket is neutrally stable. A few instants later, the C.G. is ahead of the C.P. and the rocket is in a stable condition. It is impossible to predict in just what direction the rocket will be pointing when it becomes stable and clearly, a potential hazard exists if the rocket was pointing horizontally or at a down angle.

What if this same flight was instead performed with the recommended lightest engine? Referring to the Javelin drawing again, it is noted that the rocket's C.G. with a ½A engine is ahead of the assumed C.P. to start with (9.40 inches from the nose tip). This marginal stability would not really be sufficient by our standard one body diameter stability criteria, but definitely must be considered a safer procedure than using the heavier B motor in an initial flight.

Another situation that can give rise to the unstable-to-stable flight condition is in multi-staging where the lift-off configuration is unstable, but the second stage is stable. You cannot just check the C.P. and C.G. of the second stage and conclude that the rocket will be stable. The second stage combined with its booster is an entirely different aerodynamic shape and has its own special center of pressure location. To be a stable body in free flight this C.P. must be behind the balance point of the entire two-stage vehicle.

Anyway, the only reason these rare situations can possibly occur is if the rocketeer is not conscientious and does not bother to check his C.P. and C.G. locations. Simple methods now exist to accurately check stability with confidence -- use them!

We at Centuri sincerely hope that the material presented in this report has helped to give you an appreciation and understanding of the importance of stability and along with that some motivation to use the scientific tools available at your disposal.

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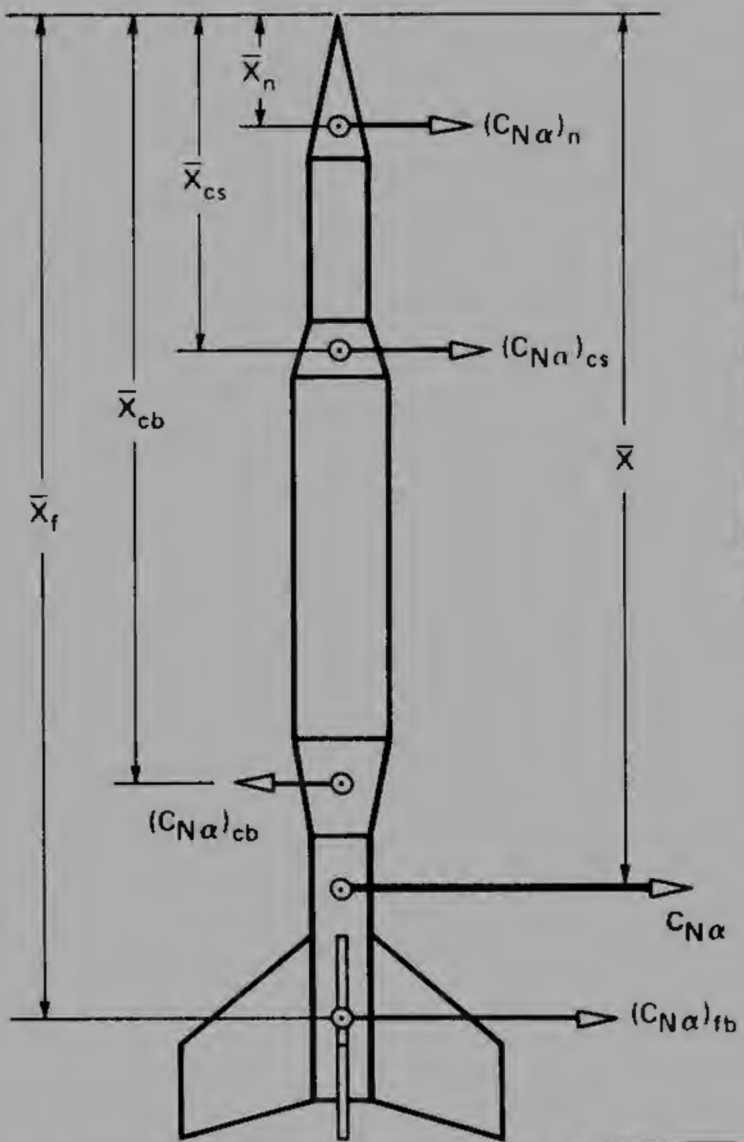
TECHNICAL INFORMATION REPORT



CALCULATING THE CENTER OF PRESSURE

OF A MODEL ROCKET

Centuri

CENTURI ENGINEERING COMPANY · BOX 1988 · PHOENIX, ARIZONA 85001

CALCULATING THE CENTER OF PRESSURE OF A MODEL ROCKET

WRITTEN BY

JAMES BARROWMAN

TABLE OF CONTENTS

Centuri

ENGINEERING COMPANY, INC.
Phoenix, Arizona

PROLOGUE

This report presents an easy to use method for accurately calculating the exact center of pressure of a subsonic rocket flying at small angles-of-attack.

The basic equations for determining the center of pressure were theoretically derived by the author for a research and development project which was presented at the National Association of Rocketry Annual Meet (NARAM-8) in August of 1966. The equations in this report were subsequently published by the Educational Services Offices of NASA in the widely distributed Information For You pamphlet entitled, "Calculating the Center of Pressure of a Rocket".

The report herein is a significant improvement over the NASA pamphlet in that all the complex equations have been reduced to chart form and all the formerly prevalent math operations such as squaring and taking square roots have been completely eliminated. The number of arithmetic operations and the corresponding chances of making mathematical mistakes as a result have been greatly reduced.

While some insight into center of pressure, center of gravity and stability criteria have been included in this report a more thorough presentation treating these subjects separately has been prepared and is available as CENTURI's Technical Information Report TIR-30.

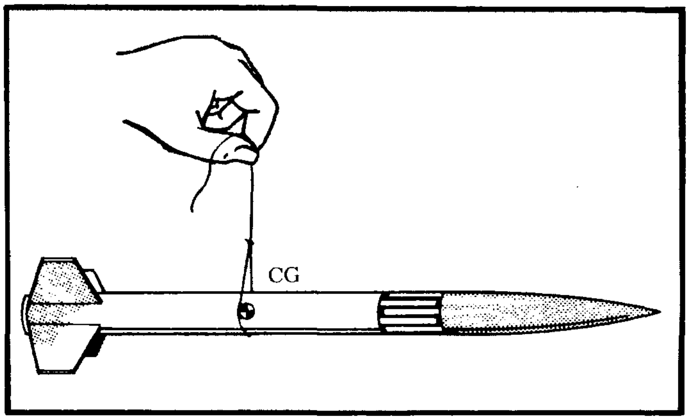
INTRODUCTION

For your model rockets to have safe, predictable flights, they must be stable. A model rocket will be stable only if its center of pressure is behind its center of gravity. Before you fly any model rocket, you must be sure that it is stable. This obviously boils down to finding the locations of the rocket's center of gravity (C.G.) and center of pressure (C.P.).

CENTER OF GRAVITY

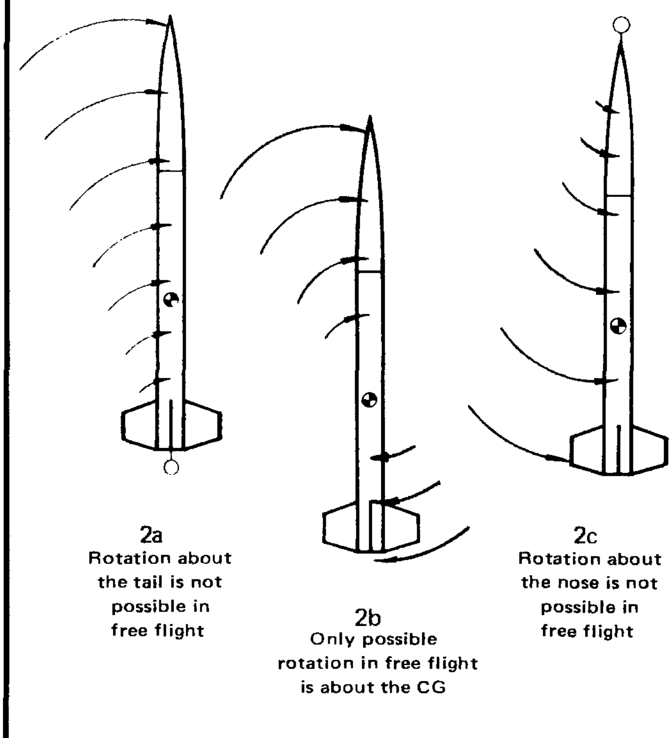
The center of gravity of a rocket is the point at which all the weight of the rocket seems to be concentrated. That is, there is as much weight distributed ahead of the rocket's center of gravity as there is behind it. Another name for the C.G. is the rocket's balance point. Finding the center of gravity of a rocket that is already built is very simple. It involves balancing the rocket with the engine and parachute inserted on a string and marking the location of the string where the rocket stays level.

FIGURE 1



The engine used, of course, should be the most powerful with the longest delay that is ever expected to be flown in the rocket. This results in the maximum weight at the back and shifts the C.G. back as far as possible.

If you are designing a rocket and want to find its C.G. location without building it, you must calculate the C.G. from a knowledge of the weights of its component parts (body tube, nose cone, engine, etc.). This, too, is relatively easy. A technique for predicting a rocket's C.G. location before it actually is built is given in Appendix A. The center of gravity (C.G.) is important to stability not because the rocket balances there; but because when the rocket wobbles in free flight, it will rotate only about the center of gravity (C.G.).



Rotation about the center of gravity

FIGURE 2

Note that the symbol ⊕ is used to represent the center of gravity on drawings.

CENTER OF PRESSURE

The center of pressure is similar to the center of gravity except that the forces involved are the air pressure forces acting on the rocket while it is flying. A formal definition is then -- The center of pressure of a rocket is the point at which all the air pressure forces on the rocket seem to be concentrated. That is, there is as much air pressure force distributed on the rocket ahead of the center of pressure as there is behind it. In the figure below, the size of the air pressure forces that are distributed over the length of the rocket and on the fins are represented by the length of the arrows along the side of the rocket.

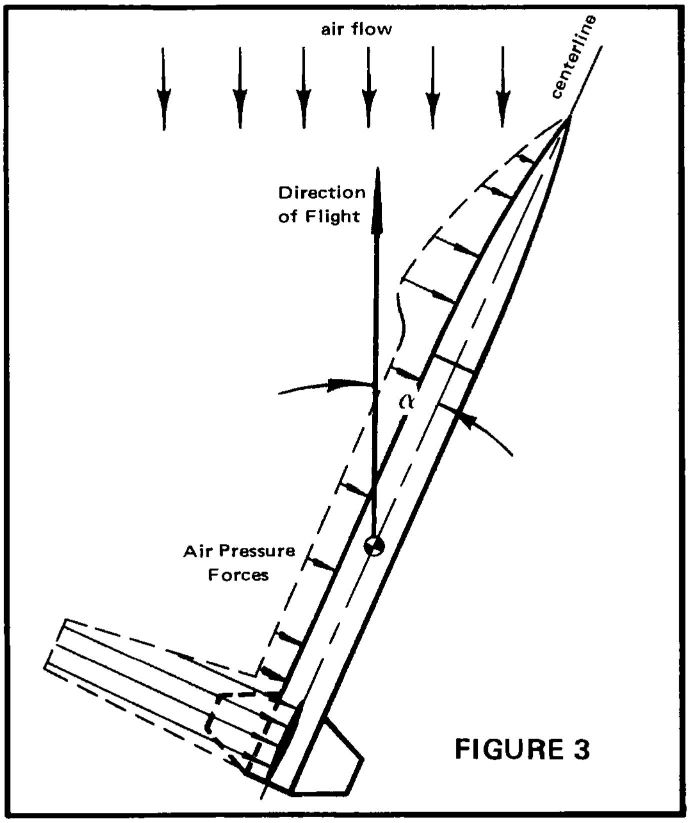


FIGURE 3

As you can see, the rocket is at an angle (highly exaggerated in the above figure) to the direction it is flying. As a result, it is at an angle to the direction of the air flow over it. This is called the angle-of-attack and is represented by the Greek letter alpha, α.

As long as the rocket isn't disturbed it will fly straight into the air flow. Now if the rocket is disturbed say by a wind gust, thrust misalignment, or cocked fin, then it will fly at an angle-of-attack. Then a stable model rocket will make continual corrections during its flight as it attempts to return to zero, just as a person manually makes constant corrections to maintain a straight path when driving a car or riding a bicycle.

Notice that the air pressure forces pictured above are all perpendicular to the rocket centerline. These are called the normal (mathematical term meaning perpendicular) forces acting on the rocket. There are also axial forces on the rocket which act parallel to the centerline (or axis) of the rocket. Although the axial air pressure forces (commonly called aerodynamic drag) are important in calculating the altitude performance of a rocket, they are not at all important in determining its center of pressure and resulting stability. If you are also interested in the affects of the axial air pressure forces on the flight of your rockets, we suggest that you study CENTURI's TIR-100 on model rocket performance.

The distribution of normal forces shown above represents how the forces actually act on a typical model rocket that is flying at an angle-of-attack (α).

However, since there is a point (the center of pressure) along the length of the rocket where there is as much normal force ahead of it as there is behind it, all the forces that are distributed along the length of the rocket can be added up into a single force that acts only at the center of pressure. The symbol ⊙ will be used in the drawings to represent the location of the center of pressure.

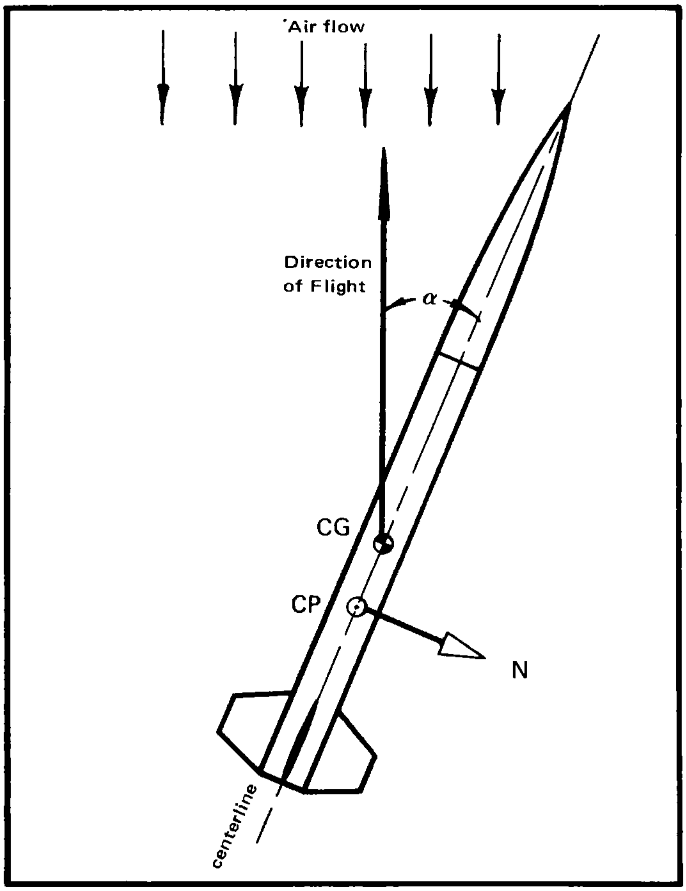


FIGURE 4

This sum of all the distributed normal forces is called <u>the</u> normal force and is represented by the letter N. The normal force, N, is the force which brings the rocket back to zero angle-of-attack and it will be discussed more thoroughly in a later section.

The angle-of-attack at which a rocket flies has a strong effect on the size and shape of the normal force distribution on the rocket. In turn, the shape of the normal force distribution determines the C.P. location. It has been found that the C.P. moves forward as the angle-of-attack increases. This fact is very important since it can affect a rocket's stability.

STABILITY CRITERIA

Knowing that a rocket is stable is not enough. You must also know how much stability the rocket has. The farther the rocket's C.P. is behind its C.G. the more stable the rocket will be. This is because the aerodynamic normal force which is pushing at the C.P. location has a longer lever arm distance to the rocket's pivot point (the C.G.) and so can return it to zero α proportionally faster. This distance between center of pressure (C.P.) and the center of gravity (C.G.) is called the static margin.

Thus, the larger its static margin the more stable it will be providing, of course, that the rocket's C.P. is behind its C.G.

The importance of the effect of the angle-of-attack (α) on C.P. location now becomes apparent. As a rocket's angle-of-attack increases; the C.P. moves forward; the static margin decreases; and therefore the rocket becomes less stable. It is possible that the C.P. might even move forward of the C.G. causing the bird to become unstable and do flip-flops.

Obviously, you want your rockets to fly at as small an angle-of-attack as possible. Not only does it insure stability but it improves altitude performance as well, since aerodynamic drag increases proportionally as angle-of-attack increases. Drag is a minimum at zero angle-of-attack. At first glance, then, it appears that the maximum static margin is desirable. However, there is one additional factor to be considered. If a rocket has a high static margin it can actually be too stable for windy-day flying. The flight path will not be anywhere near vertical but instead the rocket will consistently arc over and head into the wind. This phenomena is called weathercocking and the reasons why it occurs are covered in detail in CENTURI's TIR-30 on stability.

It turns out that a good rule of thumb developed over the years is simply to be sure that the static margin is at least equal to and preferably is just somewhat greater than the largest diameter of the rocket as shown. Model rocketeers refer to this reference diameter as the caliber of their rocket and so when you hear someone talking about "one caliber stability", you will know that the C.P. is one maximum body diameter behind the C.G.

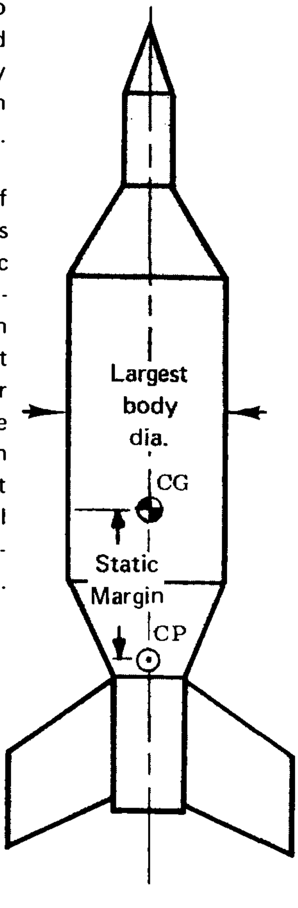


FIGURE 5

Largest body diameter = Static Margin

One caliber static margin stability insures good safe flight characteristics and at the same time it minimizes the effects of crosswinds and gusts.

The remainder of this report is devoted to presenting an accurate method for finding the center of pressure of any rocket when it is flying near zero angle-of-attack (α). The static margin of your original model rocket designs can then be verified with confidence prior to flight. The additional effort expended in performing a center of pressure analysis is flight insurance and protection for the total investment in time, effort and money spent in the construction of that super bird.

3. ELEMENTS OF THE THEORETICAL CENTER OF PRESSURE CALCULATIONS

ASSUMPTIONS

The assumptions that we used in finding any equations are very important since the assumptions indicate exactly what the mathematical equations can and cannot physically simulate. The basic assumptions used in deriving the equations in this report are as follows:

1) The angle-of-attack of the rocket is near zero (less than 10^{o}).

2) The speed of the rocket is much less than the speed of sound (not more than 600 feet per second).

3) The air flow over the rocket is smooth and does not change rapidly.

4) The rocket is thin compared to its length.

5) The nose of the rocket comes smoothly to a point.

6) The rocket is an axially symmetric rigid body.

7) The fins are thin flat plates.

Although some of the above assumptions seem quite restrictive, the vast majority of model rockets conform to these requirements. However, before analyzing your rocket, you must be sure that it is not one of the few that violate these assumptions. For example, you cannot analyze a boost glider using the equations in this report. A boost glider violates assumptions 3), 4), 5), 6), and at times 1).

NORMAL FORCE TERMINOLOGY

At angles-of-attack near zero (assumption 1) the normal force acting on a rocket depends on the shape of the rocket, the density of the air, the velocity, the size of the rocket, and the angle-of-attack. In equation form:

$$N = C_{N\alpha}\ \tfrac{1}{2}\rho V^2 \alpha A_r$$

Where:

- N is the total normal air pressure force acting on the rocket.
- $C_{N\alpha}$ is the dimensionless coefficient that accounts for the shape of the rocket.
- ρ the Greek letter rho is the density of the air.
- V is the rocket's velocity or speed. V^2 means velocity is squared or multiplied by itself.
- A_r is a reference area that indicates the size of the rocket. The reference area generally used is the cross-sectional area at the base of the nose.
- α the Greek letter alpha is the angle-of-attack.

It can be seen from the above formula that the total normal aerodynamic force (N) is larger, as expected, for larger rockets because the reference area (A_r) will be correspondingly larger. Also we note from the formula that when the angle-of-attack (α) is zero that there is no normal force (N). Similarly, the normal force (N), when α is 2 degrees, is twice as large than if α was 1 degree. Thus at a given velocity, as the angle-of-attack becomes larger the tendency of a stable rocket to realign itself to zero angle-of-attack is increased.

Another important influence on the magnitude of the normal force is the velocity of the rocket. The normal force (N) is seen to be proportional to the square of velocity (V^2). This means that doubling the velocity gives a rocket four times the force tending to return it to zero from a given disturbed angle-of-attack ($2^2 = 4$), while tripling the velocity gives nine times the correcting force ($3^2 = 9$).

This is the reason why model rockets are designed to really scoot off the launch pad and never lift off in the grand stately slow grace of a Saturn booster. Once the model rocket leaves the launch rod it is in free flight and the aerodynamic normal forces acting at the C.P. must at this time be of reasonable magnitude to provide adequate pivoting about the C.G. Obviously, the higher the rocket's velocity as it leaves the launch rod the better. This, of course, presumes the rocket is stable and has the C.P. at least one maximum body diameter (caliber) behind the C.G.

Both the angle-of-attack and the velocity are outside influences on the rocket. $C_{N\alpha}$, on the other hand, indicates the influence that the rocket configuration itself has on the normal force. Our concern is in verifying that the rocket has this static margin by actually calculating the center of pressure using known $C_{N\alpha}$'s for known shapes. For velocities much less than the speed of sound (assumption 2), $C_{N\alpha}$ depends only on the shape of the rocket. Since the calculation of the rocket's C.P. is a direct effect of the shape of the rocket being analyzed, then $C_{N\alpha}$ is essential and not N.

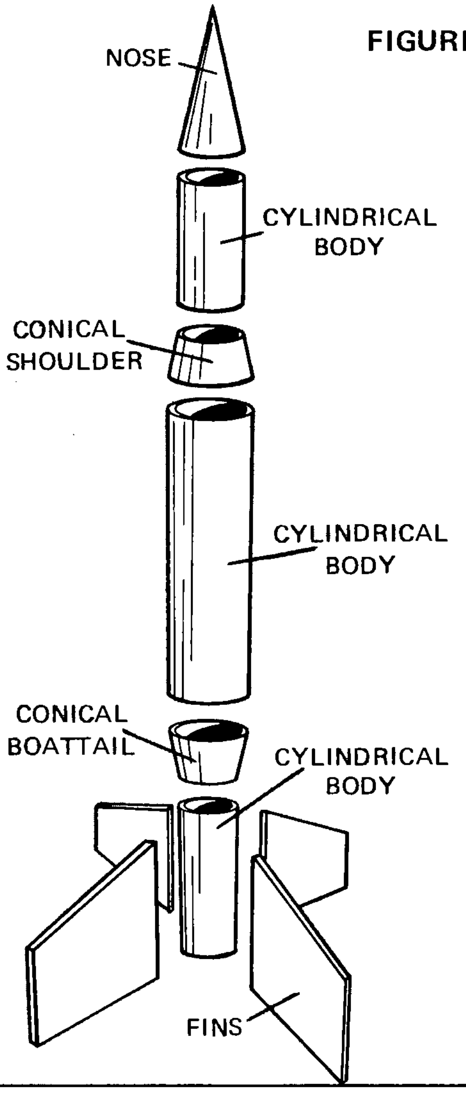
Appendix D shows why $C_{N\alpha}$ validly represents the normal force in the center of pressure calculations even though it directly is only one of the factors in determining normal forces.

For simplicity and convenience $C_{N\alpha}$ will be referred to as the normal force acting on the rocket in the rest of the report.

ANALYSIS BY REGIONS

In order to determine the center of pressure of a rocket, the rocket is divided into regions and each region is analyzed separately. Then the separate results are combined to obtain the value for the entire rocket. The particular set of equations in this paper is for a rocket that can be divided as shown in Figure 6. If there is more than one conical shoulder, conical boattail, and/or set of fins on the rocket, these should also be analyzed separately and then be included in the combination calculations.

FIGURE 6



Each region of the rocket has a normal force represented throughout the rest of the report by $C_{N\alpha}$ instead of N and a center of pressure (C.P.) associated with it just like the entire rocket. Figure 7 shows the normal forces ($C_{N\alpha}$'s) acting on the different regions of a typical rocket and their associated centers of pressure ($\overline{X}$'s) as measured from the nose tip.

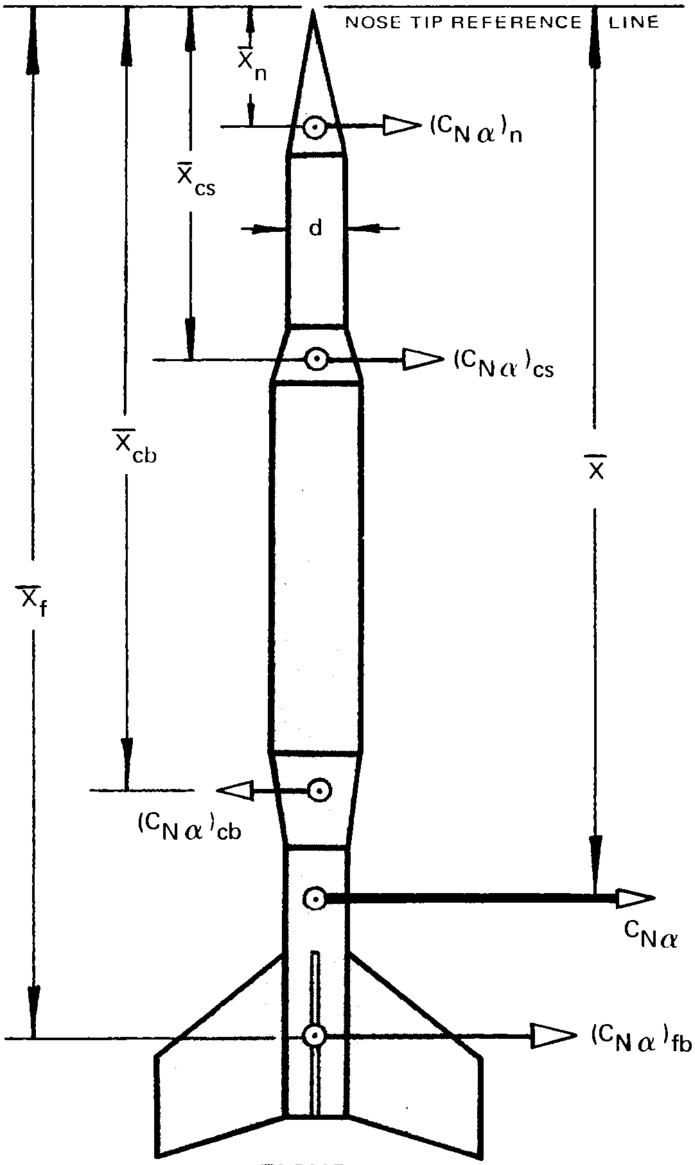


FIGURE 7
AERODYNAMIC NORMAL FORCES ACTING ON THE DIFFERENT REGIONS OF A TYPICAL ROCKET

SUBSCRIPT NOTATION

The subscripts added to $C_{N\alpha}$ and $\overline{X}$ (pronounced x bar) indicate to which part of the rocket the symbol refers. For example, the force on the nose is indicated by $(C_{N\alpha})_n$. If a symbol has no subscript, then it refers to the entire rocket. The subscripts used in this report and their meanings are as follows:

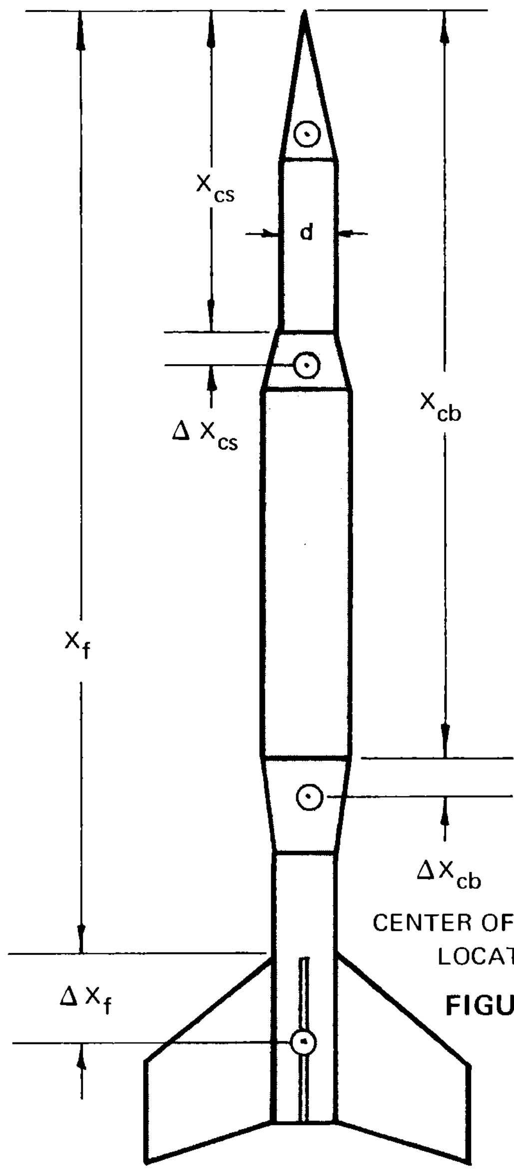
cb = Conical Boattail
cs = Conical Shoulder
f = Fins
fb = Fins in the presence of the body
n = Nose

CENTER OF PRESSURE LOCATIONS

In order to be meaningful, the center of pressure locations of all the regions of the rocket must be measured from the same reference point on the rocket. In this report the common reference point is the tip of the nose as shown below. For consistency, the C.G. should also be measured from the nose tip. Then simply subtracting the C.G. from the C.P. gives the dimension which is compared to the maximum body diameter for determining stability.

REGION LOCATIONS

In order to measure the C.P.'s of the different regions from the nose tip, the equations include the distance between the different portions and the nose tip. The definition of the symbols for the locations of the conical shoulder, conical boattail, and fins are shown in Figure 8. Notice that there are no bars above these X's. A bar is used above an X only when that X is the total distance of each region's C.P. to the reference nose tip. The small extra distance to the actual C.P. of a component part is denoted by ΔX (pronounced delta X) with the proper subscript.



CENTER OF PRESSURE LOCATIONS

FIGURE 8

REFERENCES

The equations for all the region $C_{N\alpha}$'s and C.P.'s were derived using calculus. No attempt has been made to present the actual derivations of equations as it requires a familiarity with the differentiation and integration processes of calculus. Instead, the remainder of the report simply presents the results and how to apply them to your model rocketry use.

Rocketeers who have had advanced mathematics courses should find enough information to satisfy their curiosity in the following list of references which were actually used in deriving the theoretical center of pressure equations.

1) Shapiro, A. H.; The Dynamics and Thermodynamics of Compressible Fluid Flow, Vol. 1; Ronald; New York; 1953.

2) Mayo, E. E.; Cone Cylinder and Ogive Cylinder Geometric and Mass Characteristics; Memo to Code 721.2 Files at NASA GSFC; 20 Sept. 1965.

3) Pitts, W. C.; Nielsen, J. N.; Kaatari, G. E.; Lift and Center of Pressure of Wing-Body-Tail Combinations at Subsonic, Transonic, and Supersonic Speeds; NACA TR-1307; G. P. O., Washington, D.C.; 1953.

4) Miles, J. W.; Unsteady Supersonic Flow; A. R. D. C.; Baltimore; 1955: Section 12.4.

5) McNerney, J. D.; Aerobee 350 Wind Tunnel Test Analysis; Space General Corporation; El Monte, Calif; January 1963.

6) Hoerner, Dr. S. F.; Fluid-Dynamic Drag; Midland Park, New Jersey; 1965.

4. EQUATIONS FOR FINDING THE CENTER OF PRESSURE OF THE ROCKET

The equations for each separate region are presented in the following order: nose, conical shoulder, conical boattail, and fins. The final section shows how to combine the results from all the regions into a value of $C_{N\alpha}$ and $\overline{X}$ for the entire rocket.

NOSE

In general, the normal force $(C_{N\alpha})_n$ on the nose is identical for all shapes and always has the value:

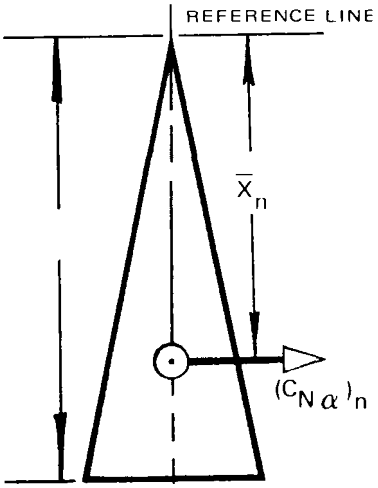
$$(C_{N\alpha})_n = 2$$

On the other hand, the center of pressure (C.P.) location on the nose varies with each different nose shape.

CONICAL NOSE

The distance from the tip of the nose to the center of pressure location of a cone-shaped nose is,

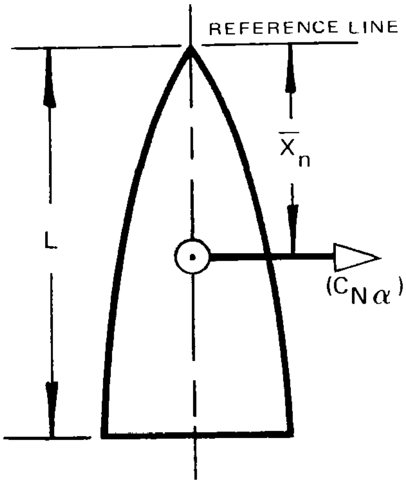
$$\overline{X}_n = \frac{2}{3}L$$



OGIVE NOSE

The distance from the tip of the nose to the center of pressure location of ogive-shaped nose is,

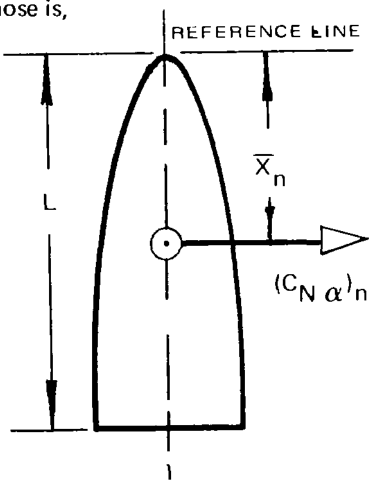
$$\overline{X}_n = .466\ L$$



PARABOLIC NOSE

The distance from the tip of the nose to the center of pressure location of a parabolic nose is,

$$\overline{X}_n = \frac{1}{2}L$$

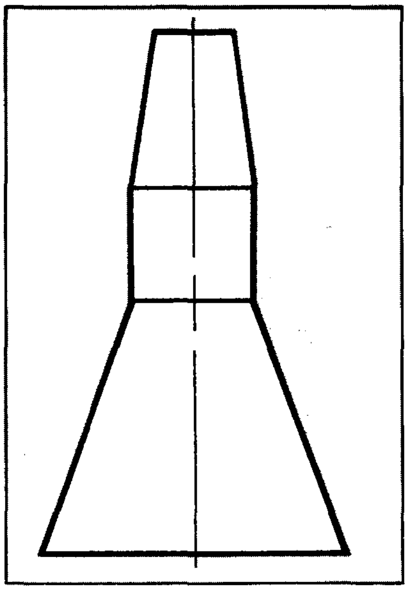


SPECIAL SHAPE NOSES

In addition to the basic shapes, there are some special frequently used nose shapes that warrant discussion.

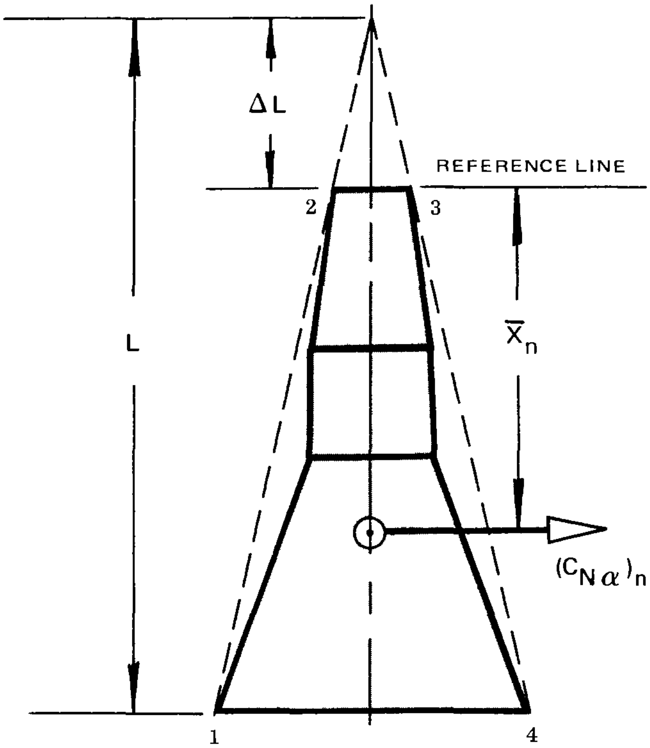
A rounded-off ogive has the same C.P. location as a parabolic nose.

The mercury capsule shape,



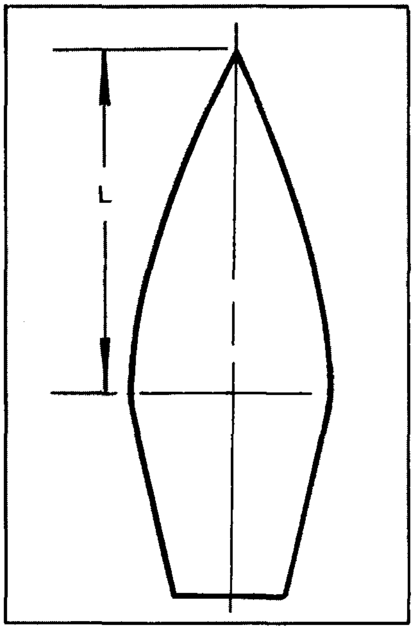
violates assumption 5 and therefore cannot be directly analyzed.

However, it has been found that such a shape can be simplified by drawing an outline of the shape and then connecting and extending its four outer-most corners (1, 2, 3, 4) to make an equivalent cone (dotted line). This technique was actually used in the initial preliminary design procedure for the Mercury, Gemini, and Apollo projects.



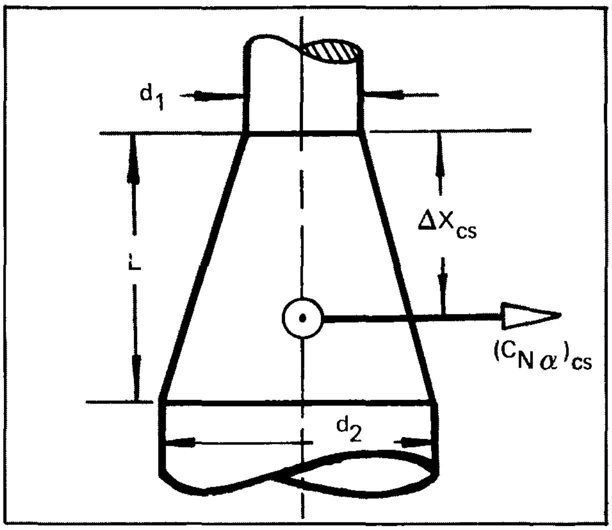
The equivalent cone is then analyzed using the equations for a cone. Remember, though, that the $\overline{X}_n$ must be calculated using the length, L, and then the length ΔL (see drawing) must be subtracted from it to give the value of $\overline{X}_n$ from the true front of the nose. This technique works for any nose shape that is similar to the Mercury capsule shape. The Gemini capsule and several other CENTURI nose-cone shapes fall into this category. CENTURI's RECRUITER kit, which has a similar nose-cone shape, will be analyzed as one of the examples in Section 8.

Another special shape is the HONEST JOHN nose cone.



Up to the largest diameter (shown by vertical dotted line), this shape is an ogive. The portion of the nose cone behind the thickest diameter can subsequently be analyzed as a conical boattail. Essentially, only the ogive portion should be considered to be the nose. This ogive is analyzed using the equation given for an ogive, using the dimension L shown as the nose length. Also the dimension "d" shown in Figure 7 should be the diameter at the base of the ogive.

CONICAL SHOULDER



The force on a conical shoulder is,

$$(C_{N\alpha})_{cs} = 2\left[\left(\frac{d_2}{d}\right)^2 - \left(\frac{d_1}{d}\right)^2\right]$$

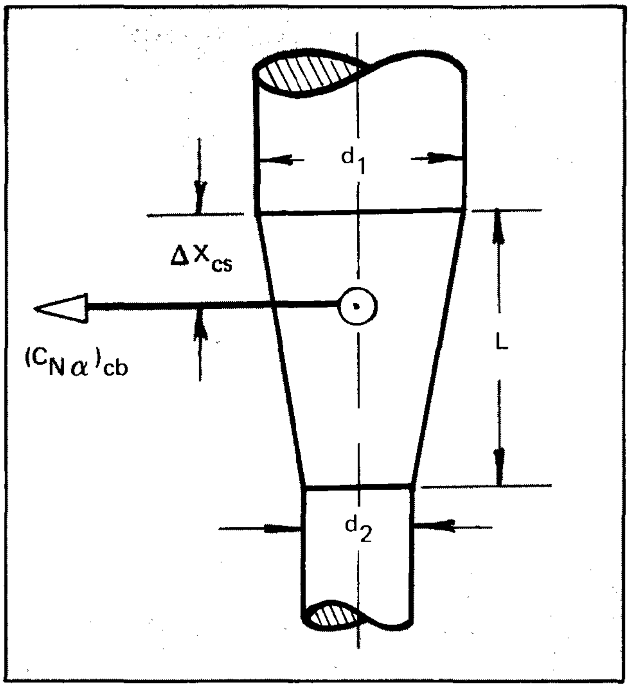
where "d" is the diameter at the base of the <u>nose.</u>

The center of pressure location of a conical shoulder is,

$$\overline{X}_{cs} = X_{cs} + \Delta X_{cs} = X_{cs} + \frac{L}{3}\left[1 + \frac{1 - \frac{d_1}{d_2}}{1 - \left(\frac{d_1}{d_2}\right)^2}\right]$$

where X_{cs} is the distance from the tip of the nose to the front of the conical shoulder (see Figure 8).

CONICAL BOATTAIL



The force on a conical boattail is,

$$(C_{N\alpha})_{cb} = 2\left[\left(\frac{d_2}{d}\right)^2 - \left(\frac{d_1}{d}\right)^2\right]$$

where "d" is the diameter at the base of the <u>nose.</u> Note that this is the same equation for the conical shoulder, however the force on a conical boattail comes out negative.

The center of pressure location of a conical boattail is,

$$\overline{X}_{cb} = X_{cb} + \Delta X_{cb} = X_{cb} + \frac{L}{3}\left[1 + \frac{1 - \frac{d_1}{d_2}}{1 - \left(\frac{d_1}{d_2}\right)^2}\right]$$

where X_{cb} is the distance from the tip of the nose to the front of the conical boattail (see Figure 8).

CYLINDRICAL BODY

For small angles-of-attack less than 10 degrees, the force on any cylindrical body portion is so small it can be neglected as can be seen from the following figure of reference 6.

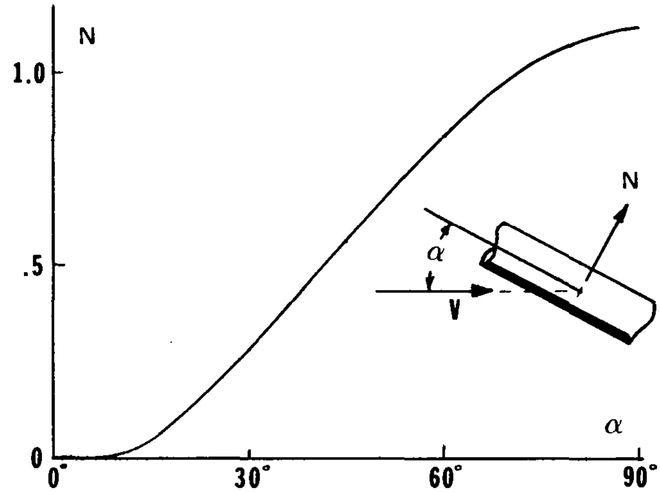
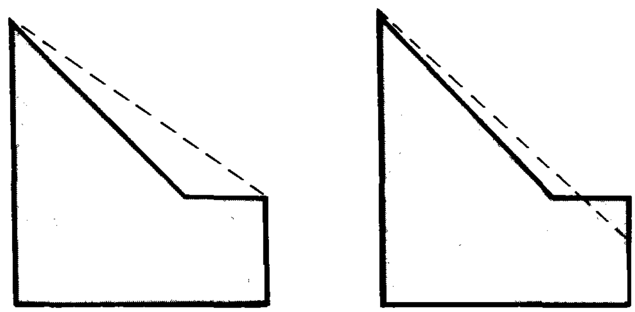


Figure 3-18 of reference 6 gives the normal force acting on circular cylinders, wires, and cables inclined to the air flow direction. This data was collected from wind tunnel tests performed primarily in the years 1918 and 1919. It is quite interesting to realize that our Space Age hobby of Model Rocketry is benefiting by engineering work done specifically to improve World War I Biplane performance.

FINS

Any fin that is not too complicated in shape may be simplified into an idealized shape that has only four straight line edges with the root and the tip parallel. It is very important when simplifying complicated fin shapes to be sure that the idealized shape has about the same or slightly less area as the actual fin. For example, Figure 9a shows an improperly idealized fin shape, while Figure 9b shows a correct simplification.

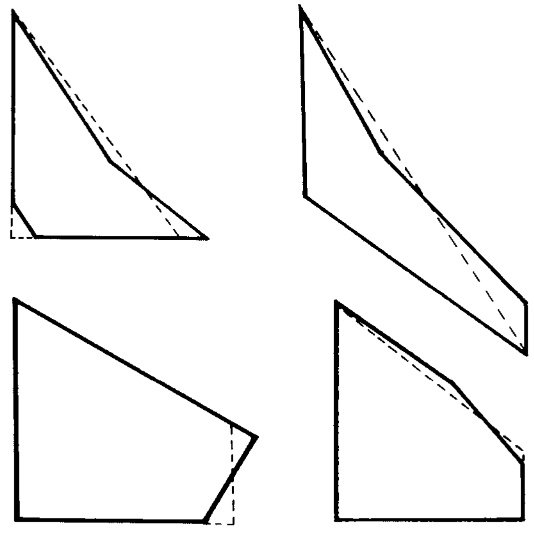


INCORRECT
BECAUSE AREA WHICH DOESN'T EXIST HAS BEEN ADDED
9a

CORRECT
BECAUSE WE STILL HAVE ABOUT THE SAME TOTAL FIN AREA
9b

FIGURE 9

The solid lines are the actual fin shape and the dotted lines show the changes made to simplify the shape. Some additional examples of properly idealized fin shapes are shown below:



OTHER IDEALIZED FIN SHAPES

FIGURE 10

The idealized fin shape and the dimensions associated with it are shown in Figure 11.

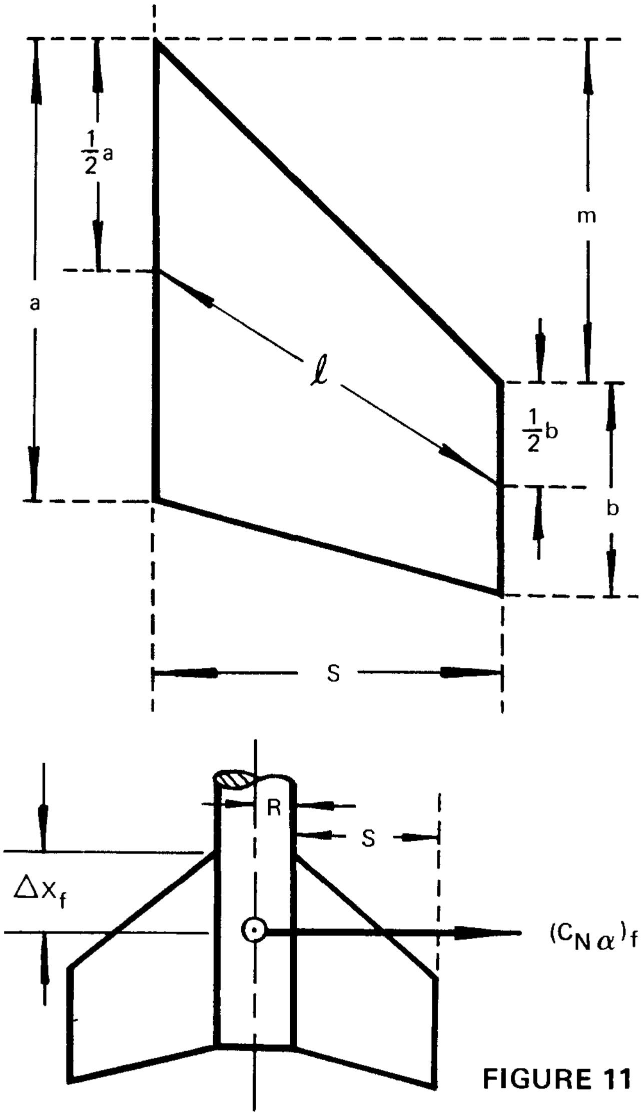


FIGURE 11

Of course, Figure 11 shows a generalized fin shape. Special cases are handled by proper use of the given dimensions. For example, a triangular fin would have b = 0. Similarly, a rectangular fin has a = b; ℓ = S; and m = 0.

In terms of the dimensions, the force on the fins of a rocket having n fins is,

$$(C_{N\alpha})_f = \frac{4n\left(\frac{s}{d}\right)^2}{1 + \sqrt{1 + \left(\frac{2\ell}{a + b}\right)^2}}$$

Where the number of identically shaped fins, n, can only be 3, 4, or 6. If your rocket has any other number of fins, these equations cannot be used. For multistage models, n refers to the number of fins on each stage. Of course, the fins on each stage must be analyzed separately and included separately in the combination equations.

FIN INTERFERENCE FACTOR

In addition, this air flow over the fins is influenced somewhat by the air flow over the body section to which the fins are attached. To account for this, the fin force for either 3 or 4 fins is multiplied by an interference factor,

$$K_{fb} = 1 + \frac{R}{S + R} \quad \text{(For n = 3 or 4)}$$

where "r" is the radius of the body between the fins and "s" is the fin semi-span shown in Figure 11. (Remember the subscript fb meant fins in presence of body). For 6 fins, however, the interference between the fins themselves cancels half the effect of the fins being attached to the body. In this case,

$$K_{fb} = 1 + \frac{.5R}{S + R} \quad \text{(For n = 6)}$$

where "r" is the radius of the body between the fins and "s" is shown in Figure 11. The total force on the tail in the presence of the body is then:

$$(C_{N\alpha})_{fb} = K_{fb}\,(C_{N\alpha})_f$$

The fin center of pressure is located in the same place on any two fins of the same size and shape. Since all the fins on a particular stage of a rocket are the same size and shape, the center of pressure location of the tail does not depend on the number of fins.

$$\overline{X_f} = X_f + \Delta X_f$$
$$= X_f + \frac{m(a + 2b)}{3(a + b)} + \frac{1}{6}\left(a + b - \frac{ab}{a + b}\right)$$

where X_f is the distance from the nose tip to the front edge of the fin root (see Figure 8).

COMBINATION CALCULATIONS

The total force on the entire rocket is the sum of all the forces on the separate regions, therefore:

TOTAL NORMAL FORCE

$$C_{N\alpha} = (C_{N\alpha})_n + (C_{N\alpha})_{cs} + (C_{N\alpha})_{cb} + (C_{N\alpha})_{fb}$$

The center of pressure of the entire rocket is found by taking a moment balance about the nose tip and solving for the total center of pressure location. (An explanation of moment balance and reasons why it works is given in Appendix B).

CENTER OF PRESSURE OF THE ENTIRE ROCKET

$$\overline{X} = \frac{(C_{N\alpha})_n \overline{X}_n + (C_{N\alpha})_{cs} \overline{X}_{cs} + (C_{N\alpha})_{cb} \overline{X}_{cb} + (C_{N\alpha})_{fb} \overline{X}_f}{C_{N\alpha}}$$

Again, remember that any additional conical shoulders, conical boattails or sets of fins must be included as extra terms in the combination equations. The additional terms fit into the combination equations in exactly the same manner as the terms for a single conical shoulder, conical boattail or fins. See the analysis of the ARCON-HI two-stage bird in Chapter 8 for an example.

Also, if the rocket you are analyzing doesn't have one of the regions included in the combination equations, then simply drop the associated term from the equations. For example, if the rocket doesn't have either a conical shoulder or a conical boattail (the JAVELIN is such a rocket, again see Section 8 on examples) the combination equations can be written:

$$C_{N\alpha} = (C_{N\alpha})_n + (C_{N\alpha})_{fb}$$

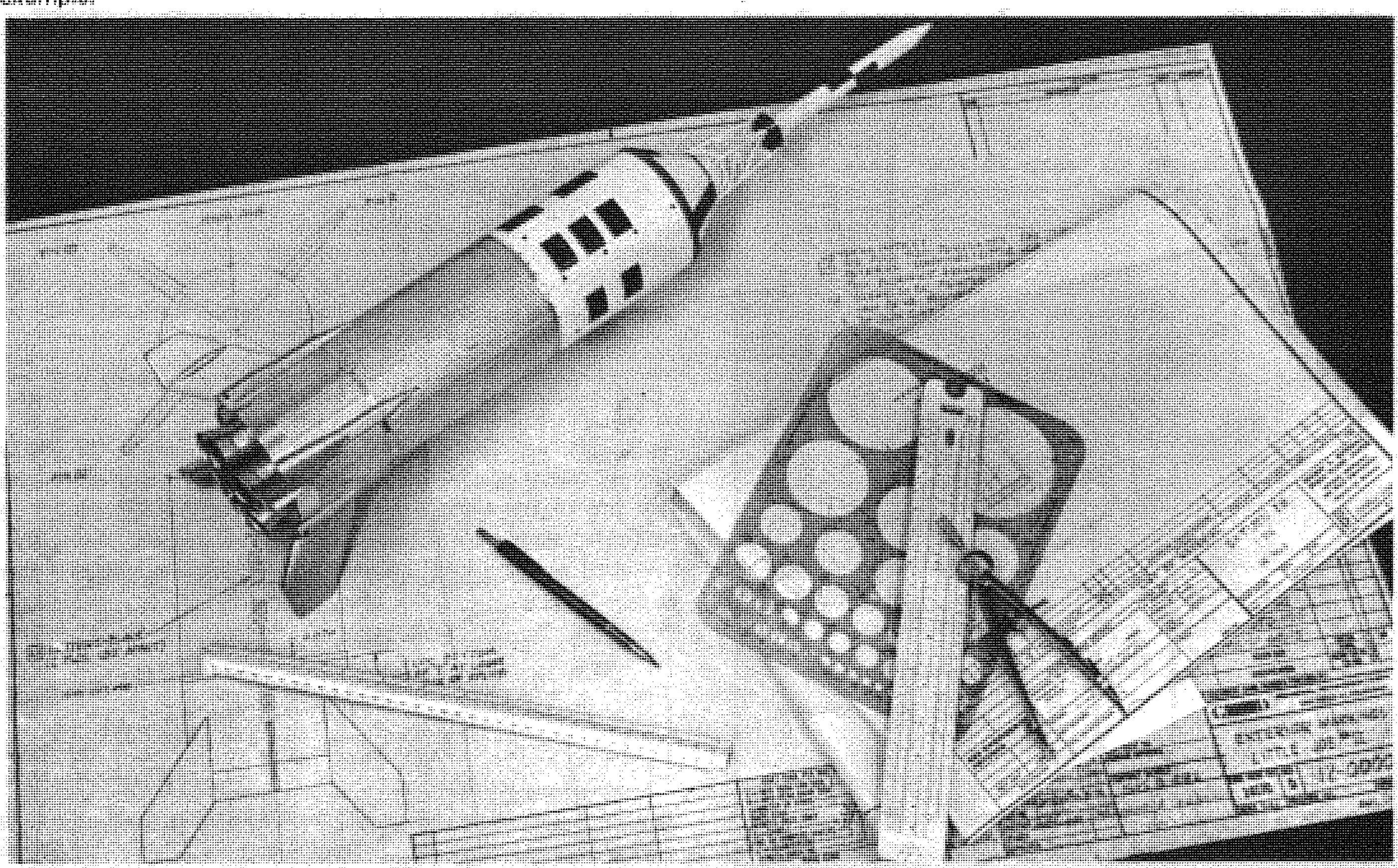
$$\overline{X} = \frac{(C_{N\alpha})_n \overline{X}_n + (C_{N\alpha})_{fb} \overline{X}_f}{C_{N\alpha}}$$

However, the above combination equations are good only for rockets having one set of fins and no shoulder or boattail.

5. SIMPLIFIED CHARTS OF THE CENTER OF PRESSURE EQUATIONS

In order to reduce the number of individual calculations, the more complex normal force and C.P. equations have been reduced to chart form. This allows even persons not well versed in reading equations to compute the center of pressure of a model rocket. In addition, it can save time and effort for those who can use the equations.

The six charts that follow give the forces and C.P. locations for conical shoulders, conical boattails, and fins alone. The nose and combination calculations are still found by using the equations. A step-by-step procedure for using the charts is given in the next section.



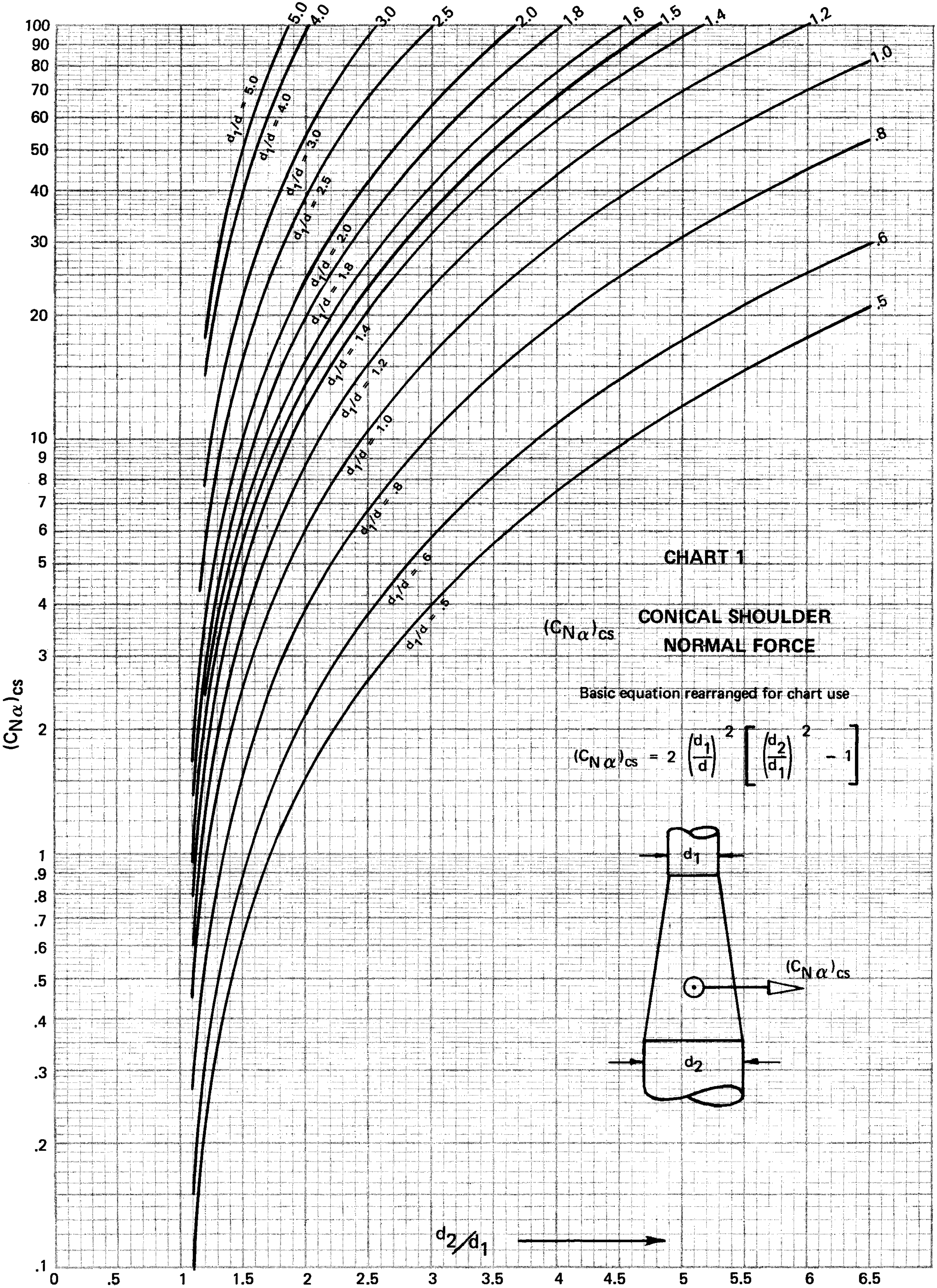


CHART 1
CONICAL SHOULDER NORMAL FORCE
$(C_{N\alpha})_{cs}$
Basic equation rearranged for chart use
$(C_{N\alpha})_{cs} = 2\left(\frac{d_1}{d}\right)^2\left[\left(\frac{d_2}{d_1}\right)^2 - 1\right]$
d_1
d_2
$(C_{N\alpha})_{cs}$
d_2/d_1
$d_1/d = 5.0$
$d_1/d = 4.0$
$d_1/d = 3.0$
$d_1/d = 2.5$
$d_1/d = 2.0$
$d_1/d = 1.8$
$d_1/d = 1.4$
$d_1/d = 1.2$
$d_1/d = 1.0$
$d_1/d = .8$
$d_1/d = .6$
$d_1/d = .5$
5.0 4.0 3.0 2.5 2.0 1.8 1.6 1.5 1.4 1.2 1.0 .8 .6 .5
100 90 80 70 60 50 40 30 20 10 9 8 7 6 5 4 3 2 1 .9 .8 .7 .6 .5 .4 .3 .2 .1
0 .5 1 1.5 2 2.5 3 3.5 4 4.5 5 5.5 6 6.5

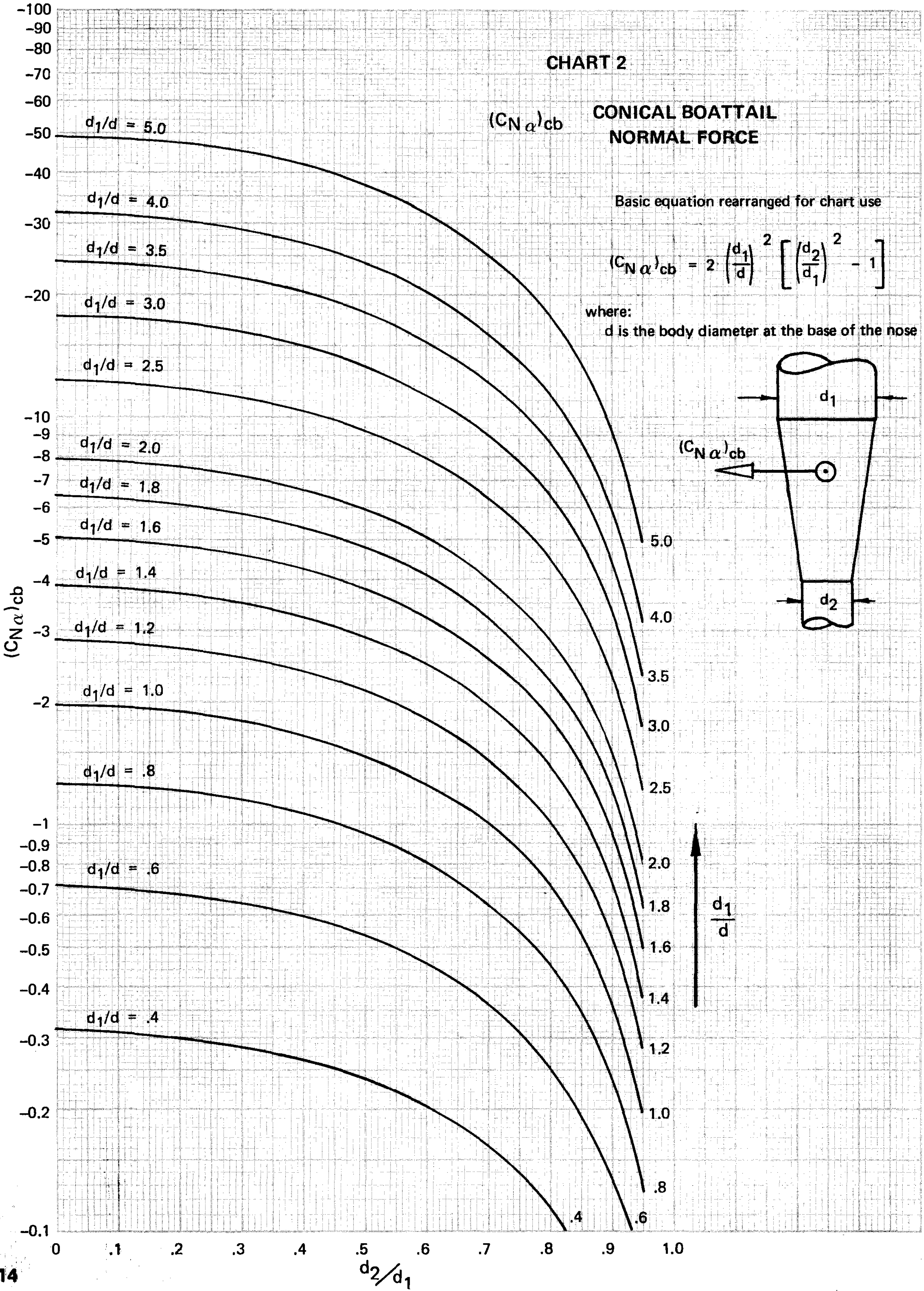


CHART 2
$(C_{N\alpha})_{cb}$
CONICAL BOATTAIL
NORMAL FORCE
Basic equation rearranged for chart use
$$(C_{N\alpha})_{cb} = 2\left(\frac{d_1}{d}\right)^2\left[\left(\frac{d_2}{d_1}\right)^2 - 1\right]$$
where:
d is the body diameter at the base of the nose
d_1
$(C_{N\alpha})_{cb}$
d_2
d_1/d = 5.0
d_1/d = 4.0
d_1/d = 3.5
d_1/d = 3.0
d_1/d = 2.5
d_1/d = 2.0
d_1/d = 1.8
d_1/d = 1.6
d_1/d = 1.4
d_1/d = 1.2
d_1/d = 1.0
d_1/d = .8
d_1/d = .6
d_1/d = .4
5.0
4.0
3.5
3.0
2.5
2.0
1.8
1.6
1.4
1.2
1.0
.8
.6
.4
$\frac{d_1}{d}$
$(C_{N\alpha})_{cb}$
-100
-90
-80
-70
-60
-50
-40
-30
-20
-10
-9
-8
-7
-6
-5
-4
-3
-2
-1
-0.9
-0.8
-0.7
-0.6
-0.5
-0.4
-0.3
-0.2
-0.1
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
d_2/d_1

CHART 3

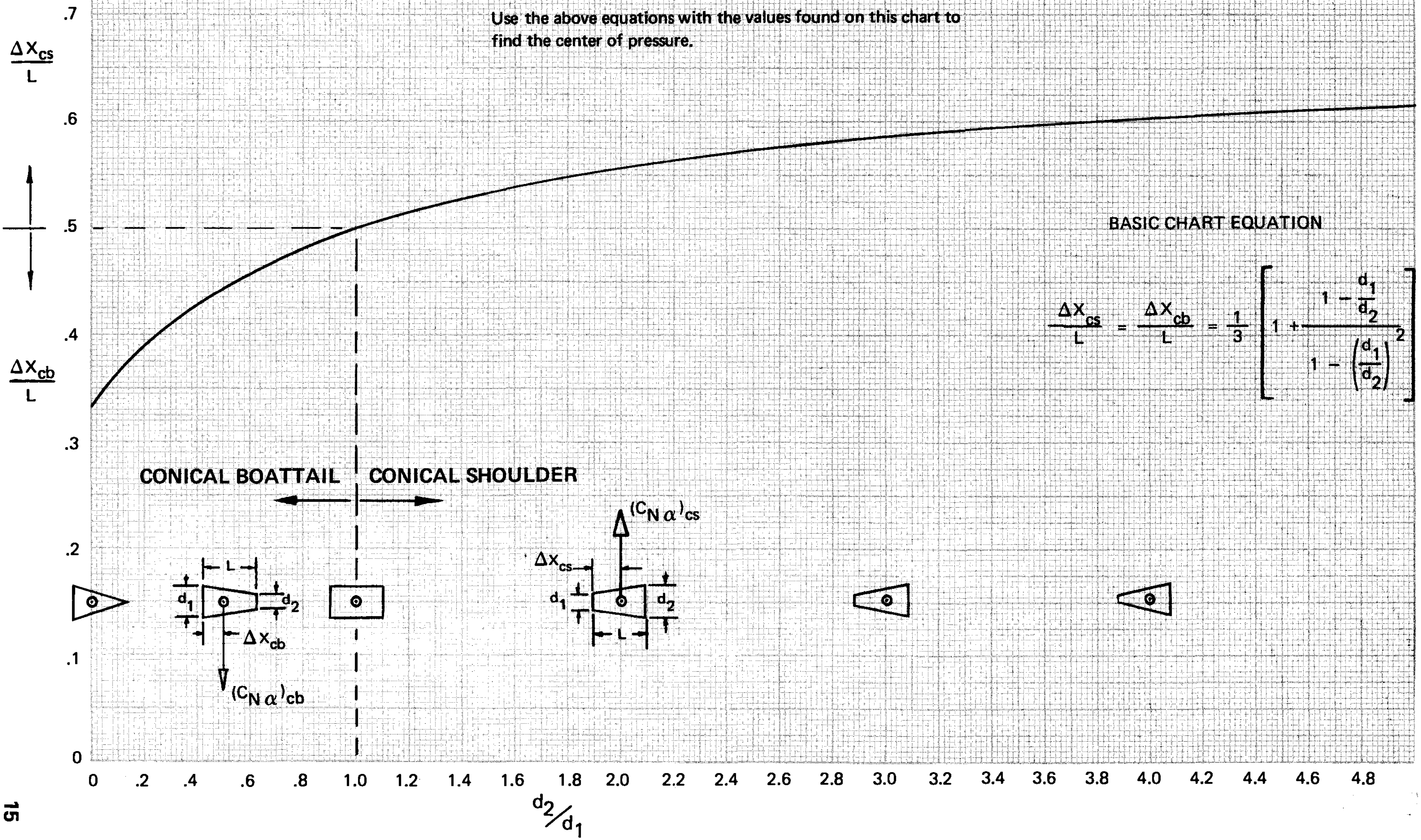
CONICAL SHOULDER AND CONICAL BOATTAIL CENTER OF PRESSURE

$$\overline{X}_{cs} = X_{cs} + \left(\frac{\Delta X_{cs}}{L}\right) L \quad , \qquad \overline{X}_{cb} = X_{cb} + \left(\frac{\Delta X_{cb}}{L}\right) L$$

Use the above equations with the values found on this chart to find the center of pressure.

BASIC CHART EQUATION

$$\frac{\Delta X_{cs}}{L} = \frac{\Delta X_{cb}}{L} = \frac{1}{3}\left[1 + \frac{1 - \frac{d_1}{d_2}}{1 - \left(\frac{d_1}{d_2}\right)^2}\right]$$



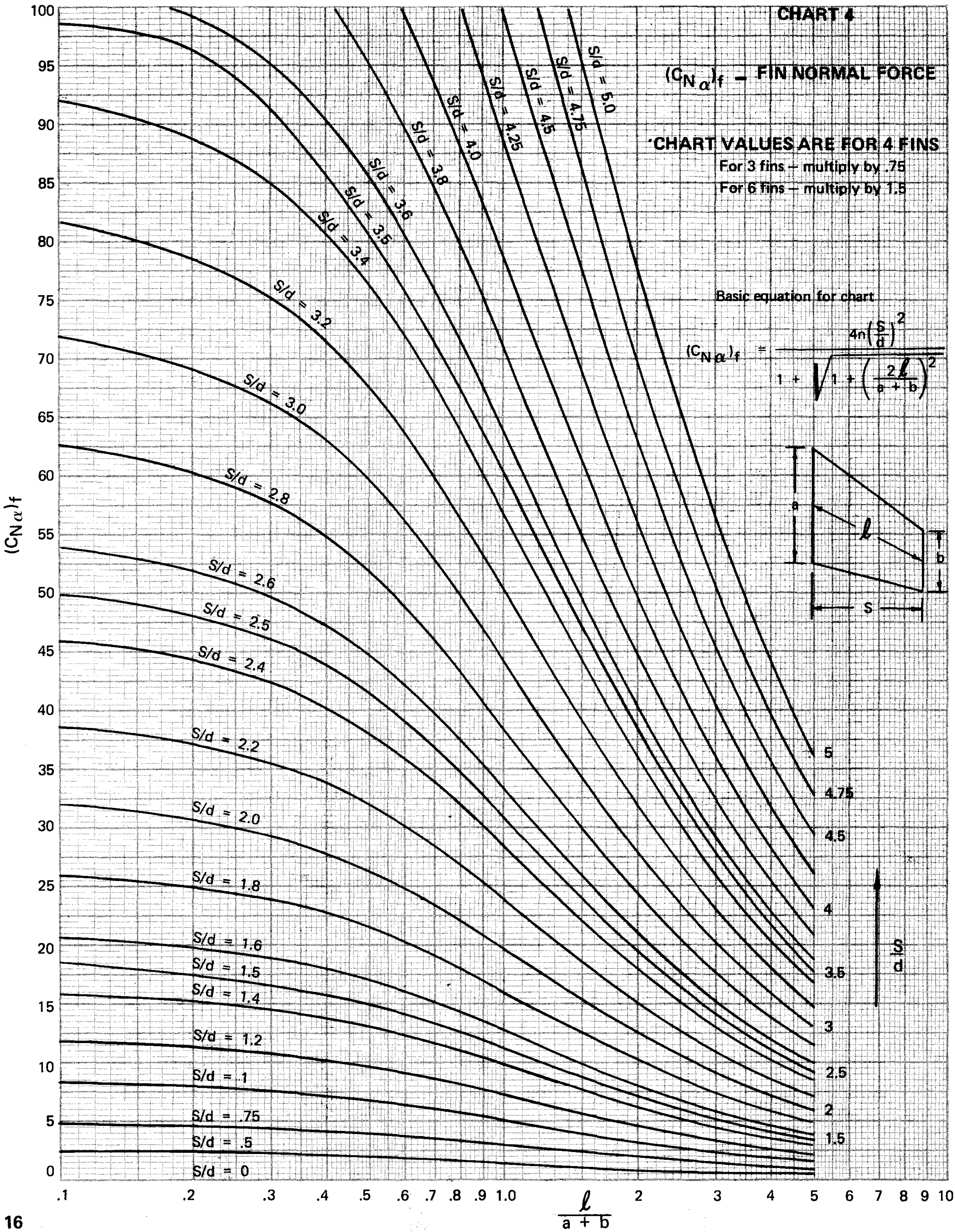
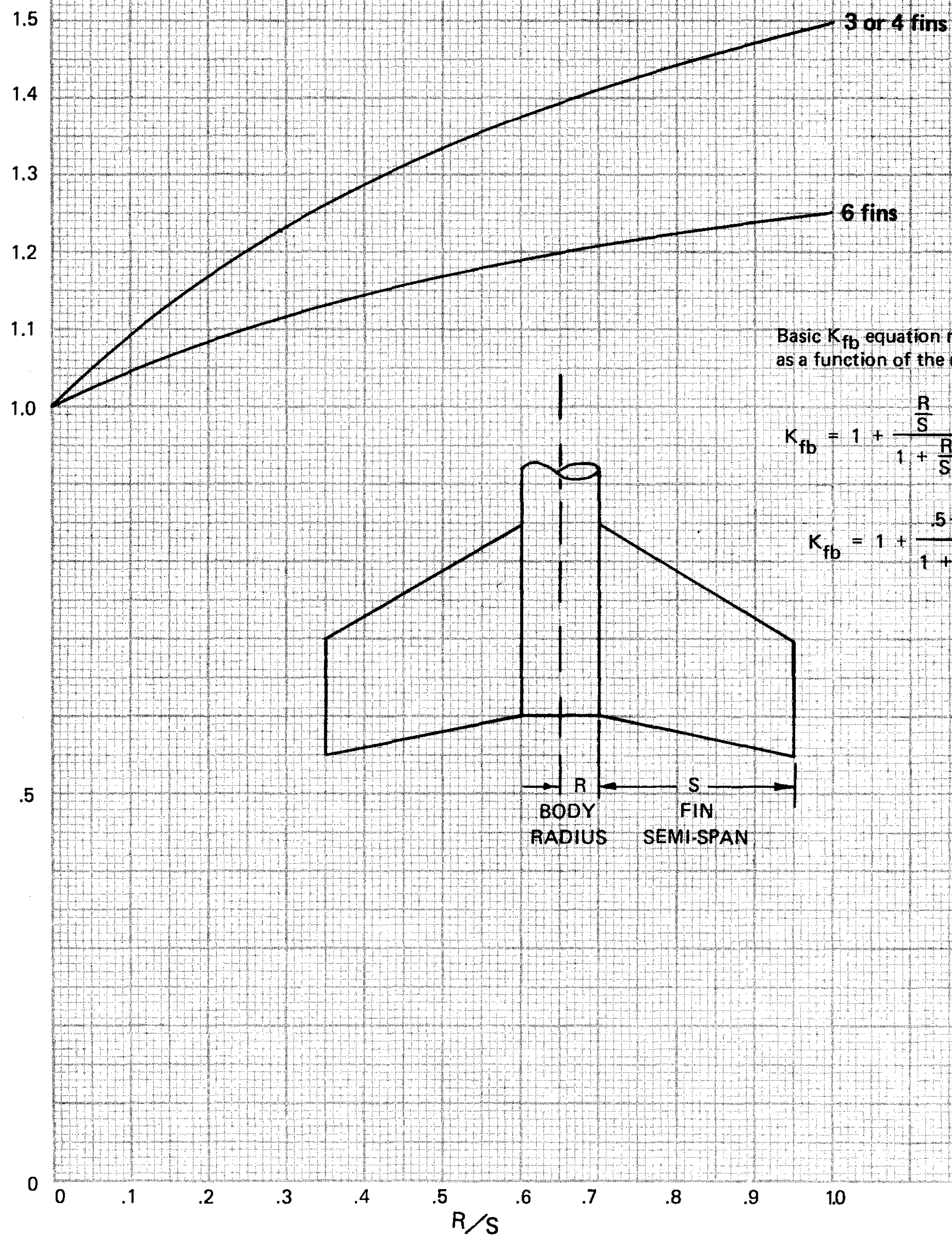


CHART 4
$(C_{N\alpha})_f$ – FIN NORMAL FORCE
CHART VALUES ARE FOR 4 FINS
For 3 fins – multiply by .75
For 6 fins – multiply by 1.5
Basic equation for chart
$$(C_{N\alpha})_f = \frac{4n\left(\frac{S}{d}\right)^2}{1 + \sqrt{1 + \left(\frac{2\ell}{a+b}\right)^2}}$$
a
b
ℓ
S
$(C_{N\alpha})_f$
$\frac{\ell}{a+b}$
$\frac{S}{d}$
S/d = 5.0
S/d = 4.75
S/d = 4.5
S/d = 4.25
S/d = 4.0
S/d = 3.8
S/d = 3.6
S/d = 3.5
S/d = 3.4
S/d = 3.2
S/d = 3.0
S/d = 2.8
S/d = 2.6
S/d = 2.5
S/d = 2.4
S/d = 2.2
S/d = 2.0
S/d = 1.8
S/d = 1.6
S/d = 1.5
S/d = 1.4
S/d = 1.2
S/d = 1
S/d = .75
S/d = .5
S/d = 0
5
4.75
4.5
4
3.5
3
2.5
2
1.5
100
95
90
85
80
75
70
65
60
55
50
45
40
35
30
25
20
15
10
5
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
2
3
4
5
6
7
8
9
10

CHART 5

K_{fb} — FIN INTERFERENCE FACTOR



Basic K_{fb} equation rearranged for plotting as a function of the ratio R/S

$$K_{fb} = 1 + \frac{\frac{R}{S}}{1 + \frac{R}{S}} \quad \text{for 3 and 4 fins}$$

$$K_{fb} = 1 + \frac{.5\frac{R}{S}}{1 + \frac{R}{S}} \quad \text{for 6 fins}$$

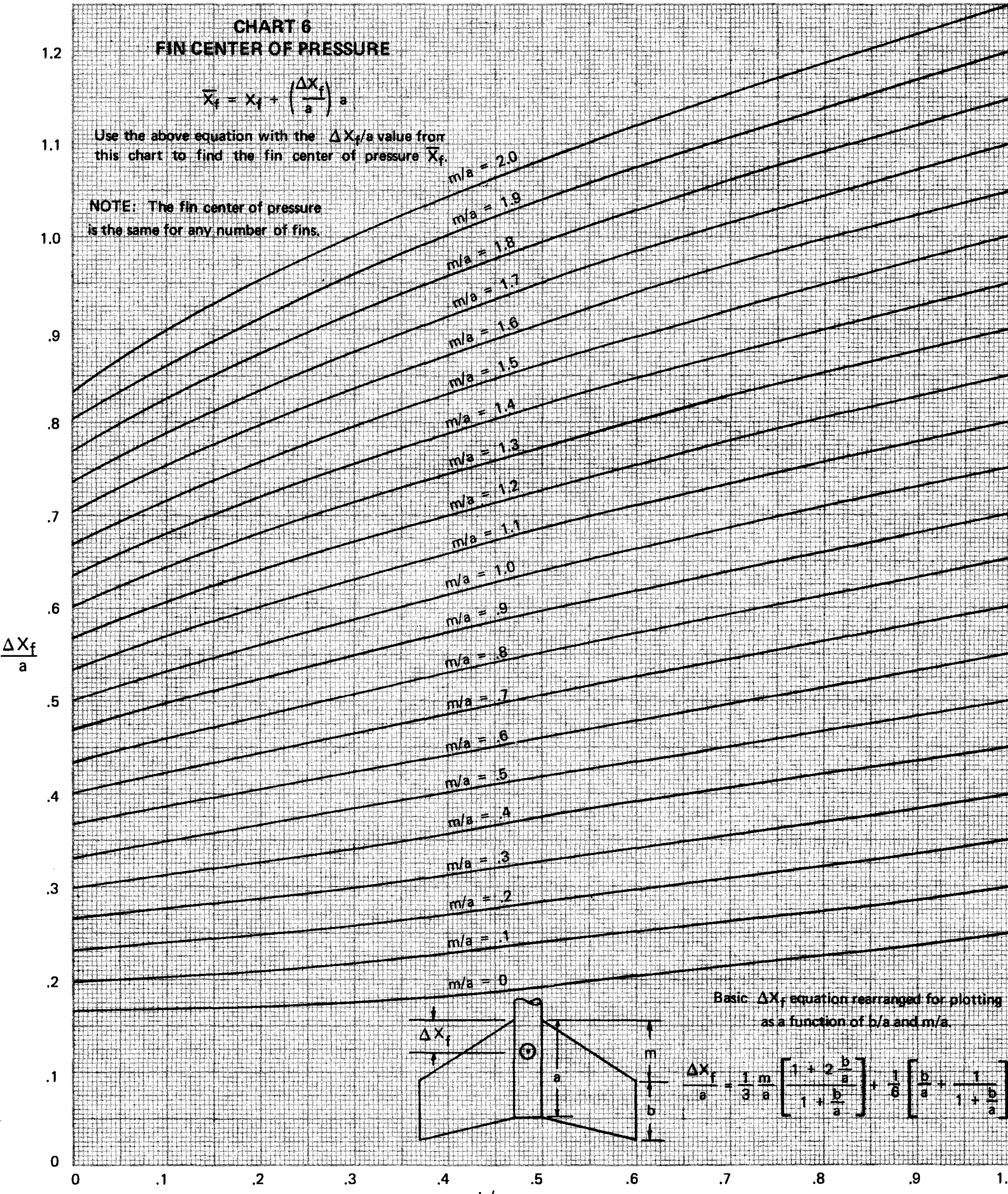


CHART 6
FIN CENTER OF PRESSURE
$\overline{X}_f = X_f + \left(\frac{\Delta X_f}{a}\right) a$
Use the above equation with the $\Delta X_f/a$ value from this chart to find the fin center of pressure $\overline{X}_f$.
NOTE: The fin center of pressure is the same for any number of fins.
m/a = 2.0
m/a = 1.9
m/a = 1.8
m/a = 1.7
m/a = 1.6
m/a = 1.5
m/a = 1.4
m/a = 1.3
m/a = 1.2
m/a = 1.1
m/a = 1.0
m/a = .9
m/a = .8
m/a = .7
m/a = .6
m/a = .5
m/a = .4
m/a = .3
m/a = .2
m/a = .1
m/a = 0
Basic ΔX_f equation rearranged for plotting as a function of b/a and m/a.
$\frac{\Delta X_f}{a} = \frac{1}{3}\frac{m}{a}\left[\frac{1+2\frac{b}{a}}{1+\frac{b}{a}}\right] + \frac{1}{6}\left[\frac{b}{a} + \frac{1}{1+\frac{b}{a}}\right]$
ΔX_f
m
a
b
$\frac{\Delta X_f}{a}$
b/a
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
1.1
1.2

6. PROCEDURE FOR USING THE CHARTS

NOSE

The nose $(C_{N\alpha})_n$ and $\overline{X}_n$ are calculated exactly as outlined in Section 4.

1. $(C_{N\alpha})_n = 2$

2. There are three basic shapes:

   a. Cone $\overline{X}_n = \frac{2}{3}L$

   b. Ogive $\overline{X}_n = .466\ L$

   c. Parabola $\overline{X}_n = \frac{1}{2}L$

CONICAL SHOULDER

1. Compute the ratios $\frac{d_1}{d}$ and $\frac{d_2}{d_1}$, then

   use Chart 1 to find $(C_{N\alpha})_{cs}$, and

   use Chart 3 to find $\left(\frac{\Delta X_{cs}}{L}\right)$

2. Compute $\overline{X}_{cs}$ using the equation

$$\overline{X}_{cs} = X_{cs} + \left(\frac{\Delta X_{cs}}{L}\right) L$$

CONICAL BOATTAIL

1. Compute $\frac{d_1}{d}$ and $\frac{d_2}{d_1}$, then

   use Chart 2 to find $(C_{N\alpha})_{cb}$, and

   use Chart 3 to find $\frac{\Delta X_{cb}}{L}$

2. Compute $\overline{X}_{cb}$ using the equation

$$\overline{X}_{cb} = X_{cb} + \left(\frac{\Delta X_{cb}}{L}\right) L$$

FINS

1. Compute $\frac{S}{d}$ and $\frac{\ell}{a + b}$, then

   use Chart 4 to get $(C_{N\alpha})_f$ for four (4) fins

   a. To convert to a three (3) fin value multiply by .75

   b. To convert to a six (6) fin value multiply by 1.5

2. Compute $\frac{R}{S}$, then

   use Chart 5 to get the interference factor K_{fb}. Be sure to use the K_{fb} line which is correct for the number of fins on the model.

   Next, compute the total force on the fins in the presence of the body

$$(C_{N\alpha})_{fb} = (C_{N\alpha})_f \ K_{fb}$$

3. Compute $\frac{m}{a}$ and $\frac{b}{a}$, then

   use Chart 6 to get $\frac{\Delta X_f}{a}$

   Compute $\overline{X}_f$ using the equation

$$\overline{X}_f = X_f + \left(\frac{\Delta X_f}{a}\right) a$$

COMBINATION CALCULATIONS

The center of pressure location of the entire rocket is calculated exactly as presented in Section 4.

$$C_{N\alpha} = (C_{N\alpha})_n + (C_{N\alpha})_{cs} + (C_{N\alpha})_{cb} + (C_{N\alpha})_{fb}$$

$$\overline{X} = \frac{(C_{N\alpha})_n \overline{X}_n + (C_{N\alpha})_{cs} \overline{X}_{cs} + (C_{N\alpha})_{cb} \overline{X}_{cb} + (C_{N\alpha})_{fb} \overline{X}_f}{C_{N\alpha}}$$

7. DESIGNING STABLE MODEL ROCKETS

Designing a new rocket is basically a problem of designing the fins. First, determine an initial rocket design that fits such requirements as the desired body tube size, payload compartment, nose-cone shape, engine type and size, and any other special features desired. Second, calculate the center of gravity (C.G.) of the design using a technique such as the one given in Appendix A. Third, calculate the center of pressure (C.P.) location of the design. Fourth, compare the result with the center of gravity to see if the design is stable and has the proper static margin. Again, a good value for the static margin is the largest body diameter of the rocket.

If the proper static margin has not been obtained, then alter the fin design and re-analyze the rocket. Changing the fin will not appreciably change the center of gravity, so only the change in center of pressure need be considered. By changing only the fin shape, only the fin terms in the combination calculations have to be changed each time.

Once the rocket is re-analyzed, check the static margin again. Keep changing the fins until the proper static margin is obtained. The changes that should be made each time will be indicated by the previous result. This is essentially a trial and error method. The more experience you have doing it, the better and faster you'll become. There are no hard and fast rules for designing anything. You must use your own engineering judgment. The center of pressure equations are just a tool to help you make judgments in deciding on a final design which will fly safely.

A few helpful hints for organizing your thoughts prior to starting calculations are given below.

1. Preliminary Work

a. Determine the needed dimensions of the rocket or proposed design.

b. Determine the nose shape (cone, ogive, parabola, or special shape).

c. Idealize the fin shape, if necessary.

d. If the nose is one of the special shapes discussed, make any drawings and/or special measurements required.

2. Calculations

a. This essentially involves plugging appropriate model dimensions into the equations and performing the mathematical operations indicated by the equations. If you are taking dimensions from a drawing rather than directly from a model, it helps to label each dimension with its appropriate terminology as done in all the examples of Section 8. It is suggested that you analyze the different regions of the rocket in the order that they are given in this report, that is:

Nose
Conical shoulder(s)
Conical boattail(s)
Fins
Combination Calculations

b. Label each set of calculations with the name of the region being analyzed. Also, circle and label the answers you get for each region. This will allow you to easily find the answers for each region and use them in the combination calculations.

c. If you can operate a slide rule, use it. Slide rule accuracy is adequate, but be sure to maintain the full 3 or 4 place accuracy available. If you are not able to operate a slide rule, learn how! It can save you very much time and effort in the long run. Like everything else worth while though, it takes some initial effort on your part to become familiar with it.

d. Above all, be neat in writing down the computations.

MULTISTAGE MODELS

If the rocket being analyzed has two or three stages, make absolutely sure that each stage is stable. For a three-stage model, you must essentially analyze three separate rockets:

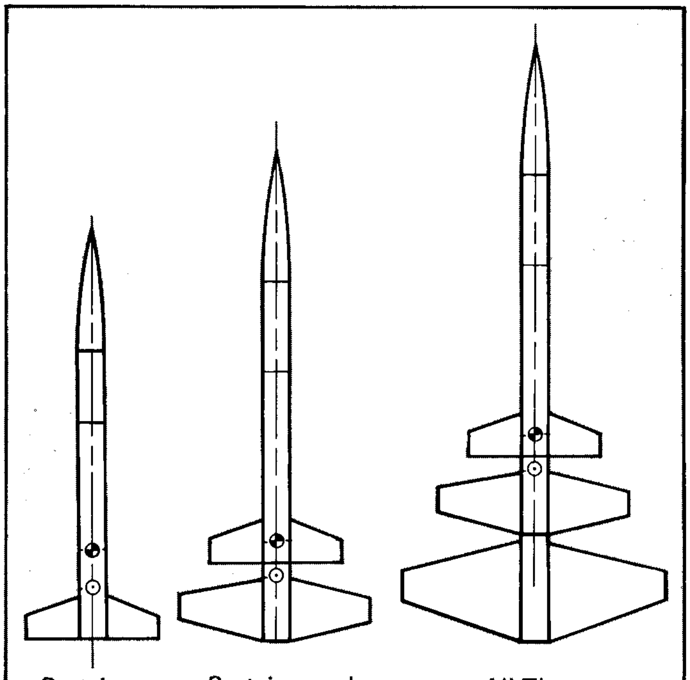


FIGURE 12

Fortunately, by analyzing the three-stage combination first, the C.P. of the other two combinations can be found simply by dropping the fin terms of the burnt-out stage from the combination calculations. An example of this technique is given for CENTURI's ARCON-HI model in Section 8. Remember! You must also find the C.G. of each stage and have the static margin of each stage equal to at least the largest body diameter.

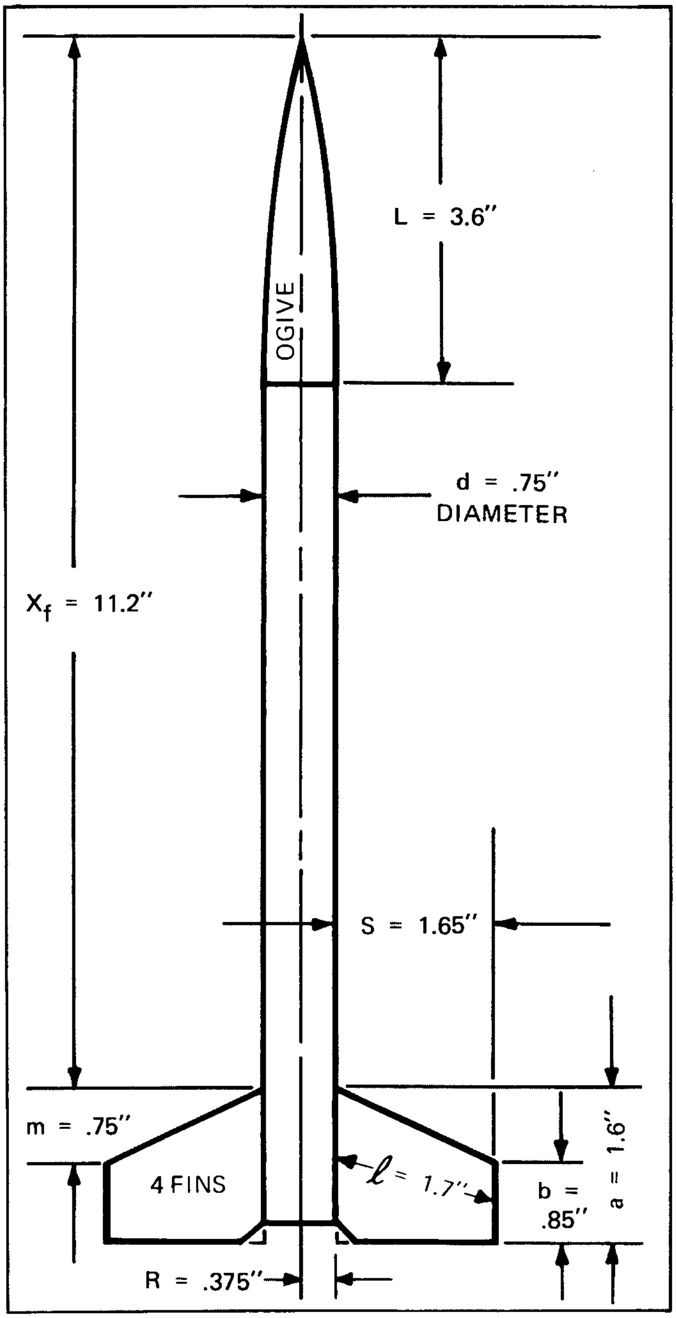
8. EXAMPLES

In order to illustrate the calculation of the center of pressure, three CENTURI model rockets are analyzed using both the equations and the charts. The three rockets that are analyzed are: the JAVELIN, a fairly simple bird with an ogive nose and one set of four fins; the RECRUITER, a more complex-shaped rocket with a complex nose shape and six fins; and the ARCON-HI, a two-stage model.

All of the mathematical operations in the following examples were done using a 10" slide rule. The numbers computed reflect the three or four significant figure accuracy obtainable with a slide rule.

JAVELIN EXAMPLE

In diagram below, the fin is idealized by including the corner at the back of the fin root (see dotted lines).



JAVELIN ANALYSIS BY USING THE EQUATIONS

Nose

normal force

$$\boxed{(C_{N\alpha})_n = 2}$$

center of pressure

$$\bar{X}_n = .466\ L$$

$$= .466\ (3.6)$$

$$\boxed{\bar{X}_n = 1.68 \text{ inches}}$$

Fins

normal force on four fins

$$(C_{N\alpha})_f = \frac{4n\left(\frac{S}{d}\right)^2}{1 + \sqrt{1 + \left(\frac{2\ell}{a+b}\right)^2}}$$

$$= \frac{4\,(4)\left(\frac{1.65}{.75}\right)^2}{1 + \sqrt{1 + \left[\frac{2\,(1.7)}{1.6 + .85}\right]^2}} = \frac{16\,(2.2)^2}{1 + \sqrt{1 + \left[\frac{3.4}{2.45}\right]^2}}$$

$$= \frac{16\,(4.84)}{1 + \sqrt{1 + (1.38)^2}} = \frac{77.4}{1 + \sqrt{2.904}} = \frac{77.4}{2.704}$$

$$\boxed{(C_{N\alpha})_f = 28.6}$$

interference factor for four fins

$$K_{fb} = 1 + \frac{R}{S + R}$$

$$= 1 + \frac{.375}{1.65 + .375} = 1 + \frac{.375}{2.025}$$

$$K_{fb} = 1.185$$

normal force on fins in presence of body

$$(C_{N\alpha})_{fb} = K_{fb}\ (C_{N\alpha})_f$$

$$= 1.185\ (28.6)$$

$$\boxed{(C_{N\alpha})_{fb} = 33.9}$$

center of pressure

$$\overline{X_f} = X_f + \Delta X_f$$

$$= X_f + \frac{m(a + 2b)}{3(a + b)} + \frac{1}{6}\left(a + b - \frac{ab}{a + b}\right)$$

$$= 11.2 + \frac{.75\ [1.6 + 2\,(.85)]}{3\,(1.6 + .85)} + \frac{1}{6}\left[1.6 + .85 - \frac{1.6\,(.85)}{1.6 + .85}\right]$$

$$= 11.2 + \frac{.25\,(1.6 + 1.7)}{2.45} + \frac{1}{6}\left[2.45 - \frac{1.36}{2.45}\right]$$

$$= 11.2 + .102\,(3.3) + \frac{1}{6}\,(2.45 - .55) = 11.2 + .34 + \frac{1.9}{6}$$

$$= 11.54 + .32$$

$$\boxed{\overline{X_f} = 11.86 \text{ inches}}$$

Javelin Combination Calculations

total normal force

$$C_{N\alpha} = (C_{N\alpha})_n + (C_{N\alpha})_{fb}$$

$$= 2 + 33.9$$

$$\boxed{C_{N\alpha} = 35.9}$$

center of pressure of the entire rocket

$$\overline{X} = \frac{(C_{N\alpha})_n \overline{X}_n + (C_{N\alpha})_{fb} \overline{X}_f}{C_{N\alpha}}$$

$$= \frac{2\,(1.68) + 33.9\,(11.86)}{35.9} = \frac{3.36 + 402}{35.9} = \frac{405}{35.9}$$

$$\boxed{\overline{X} = 11.3 \text{ inches}}$$

JAVELIN ANALYSIS BY USING THE CHARTS

Nose

(uses the equations as before)

normal force

$$\boxed{C_{N\alpha} = 2}$$

center of pressure

$$\overline{X}_n = .466\ L$$

$$= .466\,(3.6)$$

$$\boxed{\overline{X}_n = 1.68 \text{ inches}}$$

Fins

normal force

$$\frac{S}{d} = \frac{1.65}{.75} = 2.2, \quad \frac{\ell}{a + b} = \frac{1.7}{1.6 + .85} = \frac{1.7}{2.45} = .69$$

$(C_{N\alpha})_f = 28.5$ (from Chart 4)

interference factor

$$\frac{R}{S} = \frac{.375}{1.65} = .227$$

$K_{fb} = 1.185$ (from Chart 5)

total normal force on fins in presence of body

$$(C_{N\alpha})_{fb} = K_{fb}\ (C_{N\alpha})_f$$

$$= 1.185\,(28.6)$$

$$\boxed{(C_{N\alpha})_{fb} = 33.8}$$

center of pressure

$$\frac{m}{a} = \frac{.75}{1.6} = .47, \quad \frac{b}{a} = \frac{.85}{1.6} = .53$$

$\frac{X_f}{a} = .41$ (from Chart 6)

$$\overline{X_f} = X_f + \left(\frac{X_f}{a}\right) a$$

$$= 11.2 + (.41)\ 1.6 = 11.2 + .66$$

$$\boxed{\overline{X_f} = 11.86}$$

Javelin Combination Calculations

(uses the equations as before)

total normal force

$$C_{N\alpha} = (C_{N\alpha})_n + (C_{N\alpha})_{fb}$$

$$= 2 + 33.8$$

$$\boxed{C_{N\alpha} = 35.8}$$

center of pressure of entire rocket

$$\overline{X} = \frac{(C_{N\alpha})_n \overline{X}_n + (C_{N\alpha})_{fb} \overline{X}_f}{C_{N\alpha}}$$

$$= \frac{2\,(1.68) + 33.8\,(11.68)}{35.8} = \frac{3.36 + 401}{35.8} = \frac{404}{35.8}$$

$$\boxed{\overline{X} = 11.3 \text{ inches}}$$

JAVELIN STABILITY CHECK

Now that the center of pressure location has been determined, a location for the center of gravity (C.G.) can be established which will guarantee safe stable flights. For one caliber stability, the C.G. (including engine and parachute) should be one body diameter (D) ahead of the center of pressure.

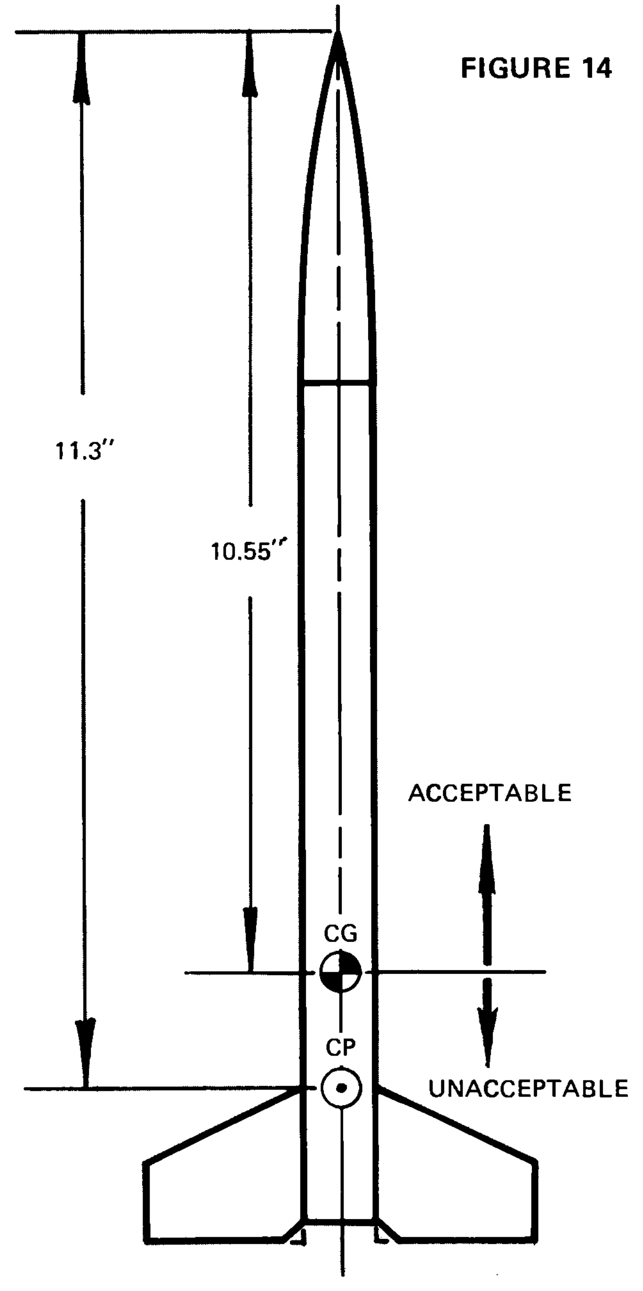
$$X_{CG} = \overline{X} - D$$

$$= 11.3 - .75$$

$$\boxed{X_{CG} = 10.55 \text{ inches from the nose tip}}$$

This C.G. value represents the furthest acceptable balance point from the nose. Balance points closer to the nose make the rocket proportionally more stable.

FIGURE 14



RECRUITER EXAMPLE

A diagram of the RECRUITER and its required dimensions is shown below.

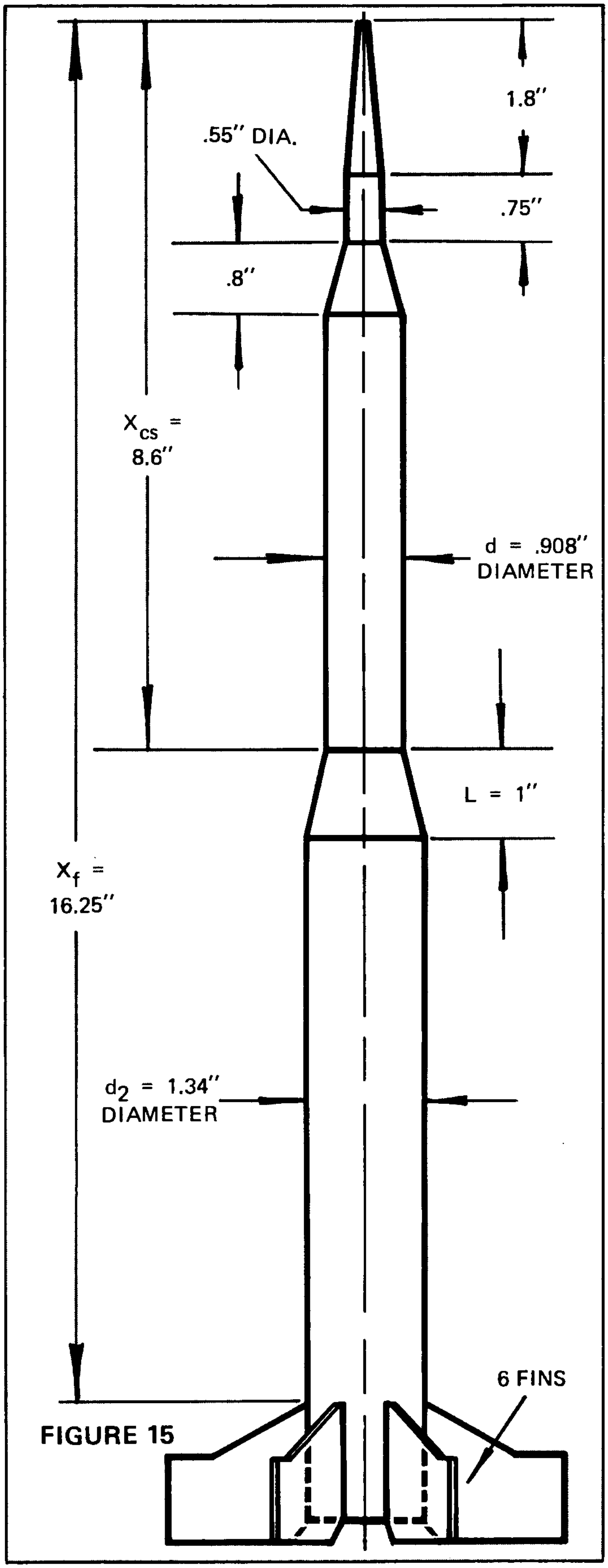


FIGURE 15

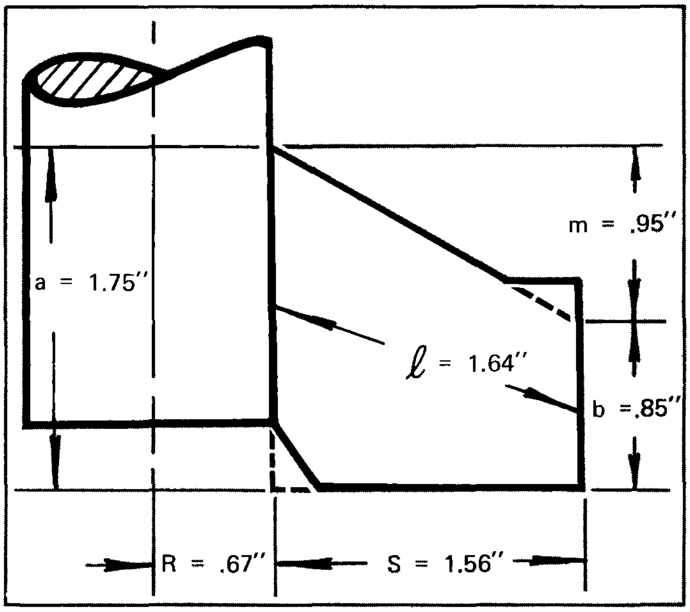
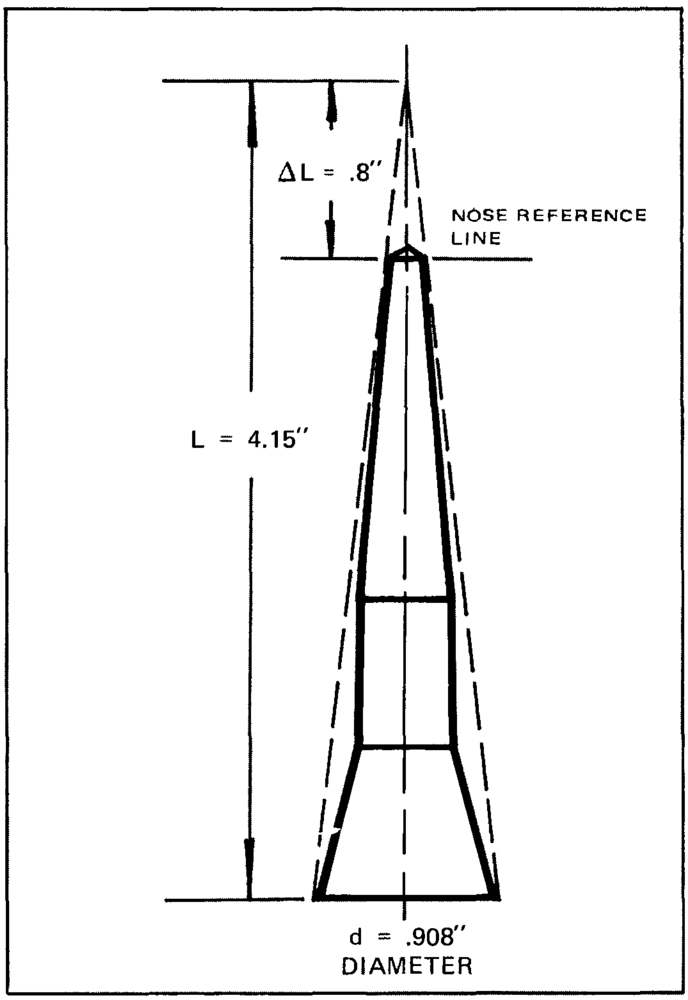


FIGURE 16

The RECRUITER's fins are idealized as shown above (actual size).

The RECRUITER's nose shape is idealized as shown below (actual size).

FIGURE 17



RECRUITER ANALYSIS BY USING THE EQUATIONS

Nose

normal force

$$\boxed{(C_{N\alpha})_n = 2}$$

center of pressure

$$\overline{X}_n = \frac{2}{3}L$$

$$= \frac{2}{3}(4.15)$$

$$\overline{X}_n = 2.77 \text{ inches}$$

But, the above C.P. is measured from the imaginary tip of the idealized cone. Correcting it back to the actual nose tip yields:

$$\overline{X}_n = 2.77 - .8$$

$$\boxed{\overline{X}_n = 1.97 \text{ inches}}$$

Conical Shoulder

normal force

$$(C_{N\alpha})_{cs} = 2\left[\left(\frac{d_2}{d}\right)^2 - \left(\frac{d_1}{d}\right)^2\right]$$

$$= 2\left[\left(\frac{1.34}{.908}\right)^2 - \left(\frac{.908}{.908}\right)^2\right]$$

$$= 2\left[(1.476)^2 - 1\right] = 2\,(2.18 - 1) = 2\,(1.18)$$

$$\boxed{(C_{N\alpha})_{cs} = 2.36}$$

center of pressure location

$$\overline{X}_{cs} = X_{cs} + \Delta X_{cs} = X_{cs} + \frac{L}{3}\left[1 + \frac{1 - \frac{d_1}{d_2}}{1 - \left(\frac{d_1}{d_2}\right)^2}\right]$$

$$= 8.6 + \frac{1.0}{3}\left[1 + \frac{1 - \frac{.908}{1.34}}{1 - \left(\frac{.908}{1.34}\right)^2}\right]$$

$$= 8.6 + \frac{1}{3}\left(1 + \frac{.322}{1 - (.678)^2}\right)$$

$$= 8.6 + \frac{1}{3}\left(1 + \frac{.322}{.541}\right) = 8.6 + \frac{1.595}{3} = 8.6 + .532$$

$$\boxed{\overline{X}_{cs} = 9.13 \text{ inches}}$$

Fins

normal force on six fins

$$(C_{N\alpha})_f = \frac{4n\left(\frac{S}{d}\right)^2}{1 + \sqrt{1 + \left(\frac{2\ell}{a + b}\right)^2}}$$

$$= \frac{4(6)\left(\frac{1.56}{.908}\right)^2}{1 + \sqrt{1 + \left[\frac{2(1.64)}{1.75 + .8}\right]^2}} = \frac{24(1.717)^2}{1 + \sqrt{1 + \left(\frac{3.28}{2.55}\right)^2}}$$

$$= \frac{24(2.95)}{1 + \sqrt{1 + (1.286)^2}} = \frac{70.8}{1 + \sqrt{2.65}} = \frac{70.8}{2.63}$$

$$(C_{N\alpha})_f = 26.9$$

interference factor for six fins

Since the RECRUITER has six fins, the interference factor is calculated using the equation having the .5 factor in the numerator.

$$K_{fb} = 1 + \frac{.5R}{S + R}$$

$$= 1 + \frac{.5(.67)}{1.56 + .67} = 1 + \frac{.335}{2.23}$$

$$K_{fb} = 1.15$$

normal force on fins in presence of body

$$(C_{N\alpha})_{fb} = K_{fb}\,(C_{N\alpha})_f$$

$$= 1.15\,(26.9)$$

$$\boxed{(C_{N\alpha})_{fb} = 31.0}$$

center of pressure

$$\overline{X}_f = X_f + \Delta X_f$$

$$= X_f + \frac{m(a + 2b)}{3(a + b)} + \frac{1}{6}\left(a + b - \frac{ab}{a + b}\right)$$

$$= 16.25 + \frac{.95\left[1.75 + 2(.8)\right]}{3(1.75 + .8)}$$

$$+ \frac{1}{6}\left[1.75 + .8 - \frac{1.75(.8)}{1.75 + .8}\right]$$

$$= 16.25 + \frac{.317(1.75 + 1.6)}{2.55} + \frac{1}{6}\left(2.55 - \frac{1.4}{2.55}\right)$$

$$= 16.25 + .124(3.35) + \frac{1}{6}(2.55 - .55)$$

$$= 16.25 + .42 + \frac{2}{6} = 16.67 + .33$$

$$\boxed{\overline{X}_f = 17.0 \text{ inches}}$$

Recruiter Combination Calculations

total normal force

$$C_{N\alpha} = (C_{N\alpha})_n + (C_{N\alpha})_{cs} + (C_{N\alpha})_{fb}$$

$$= 2 + 2.36 + 31.0$$

$$\boxed{C_{N\alpha} = 35.4}$$

center of pressure of the entire rocket

$$\overline{X} = \frac{(C_{N\alpha})_n \overline{X}_n + (C_{N\alpha})_{cs} \overline{X}_{cs} + (C_{N\alpha})_{fb} \overline{X}_f}{C_{N\alpha}}$$

$$= \frac{2(1.97) + 2.36(9.13) + 31(17)}{35.4}$$

$$= \frac{3.94 + 21.5 + 527}{35.4} = \frac{552}{35.4}$$

$$\boxed{\overline{X} = 15.6 \text{ inches}}$$

RECRUITER ANALYSIS BY USING THE CHARTS

Nose

(uses the equations as before)

normal force

$$\boxed{(C_{N\alpha})_n = 2}$$

center of pressure

$$\overline{X}_n = \frac{2}{3}L$$

$$= \frac{2}{3}(4.15)$$

$$\overline{X}_n = 2.77 \text{ inches}$$

But the above C.P. is measured from the imaginary tip of the idealized cone. Correcting it back to the actual nose tip yields:

$$\overline{X}_n = 2.77 - .8$$

$$\boxed{\overline{X}_n = 1.97 \text{ inches}}$$

Conical Shoulder

normal force

$$\frac{d_1}{d} = \frac{.908}{.908} = 1, \frac{d_2}{d_1} = \frac{1.34}{.908} = 1.48$$

$$\boxed{(C_{N\alpha})_{cs} = 2.45 \text{ (from Chart 1)}}$$

center of pressure

$$\frac{\Delta X_{cs}}{L} = .532 \text{ (from Chart 3)}$$

$$\overline{X}_{cs} = X_{cs} + \left(\frac{\Delta X_{cs}}{L}\right) L$$

$$= 8.6 + (.532)(1.0) = 8.6 + .532$$

$$\boxed{\overline{X}_{cs} = 9.13 \text{ inches}}$$

Fins

normal force

$$\frac{S}{d} = \frac{1.56}{.908} = 1.72, \quad \frac{\ell}{a+b} = \frac{1.64}{1.75 + .8} = \frac{1.64}{2.55} = .64$$

$$(C_{N\alpha})_f = 18 \text{ (four fin value from Chart 4)}$$

Since the RECRUITER has six fins, the value of $(C_{N\alpha})_f$ from the four fin chart must be multiplied by 1.5.

$$(C_{N\alpha})_f = 1.5\,(18) = 27$$

interference factor

$$\frac{R}{S} = \frac{.67}{1.56} = .429$$

$$K_{fb} = 1.15 \text{ (six fin value from Chart 5)}$$

total normal force on the fins in the presence of the body

$$(C_{N\alpha})_{fb} = K_{fb}\,(C_{N\alpha})_f$$

$$= 1.15\,(27)$$

$$\boxed{(C_{N\alpha})_{fb} = 31.0}$$

center of pressure

$$\frac{m}{a} = \frac{.95}{1.75} = .54, \quad \frac{b}{a} = \frac{.8}{1.75} = .46$$

$$\frac{\Delta X_f}{a} = .42 \text{ (from Chart 6)}$$

$$\overline{X}_f = X_f + \left(\frac{\Delta X_f}{a}\right) a$$

$$= 16.25 + (.42)\,1.75 = 16.25 + .735$$

$$\boxed{\overline{X}_f = 16.99 \text{ inches}}$$

Recruiter Combination Calculations

(uses the equations as before)

total normal force

$$C_{N\alpha} = (C_{N\alpha})_n + (C_{N\alpha})_{cs} + (C_{N\alpha})_{fb}$$

$$= 2 + 2.45 + 31.0$$

$$\boxed{C_{N\alpha} = 35.4}$$

center of pressure of the entire rocket

$$\overline{X} = \frac{(C_{N\alpha})_n\,\overline{X}_n + (C_{N\alpha})_{cs}\,\overline{X}_{cs} + (C_{N\alpha})_{fb}\,\overline{X}_f}{C_{N\alpha}}$$

$$= \frac{2\,(1.97) + 2.45\,(9.13) + 31\,(16.99)}{35.4}$$

$$= \frac{3.94 + 22.4 + 527.}{35.4} = \frac{553.}{35.4}$$

$$\boxed{\overline{X} = 15.6 \text{ inches}}$$

RECRUITER STABILITY CHECK

The C.G. location from the nose tip which will give one caliber stability is

$$X_{CG} = \overline{X} - D$$

$$= 15.6 - 1.34$$

$$\boxed{X_{CG} = 14.26 \text{ inches from the nose}}$$

Note that the diameter D is the largest diameter tube used in the construction of the model ($D = d_2$ in this instance).

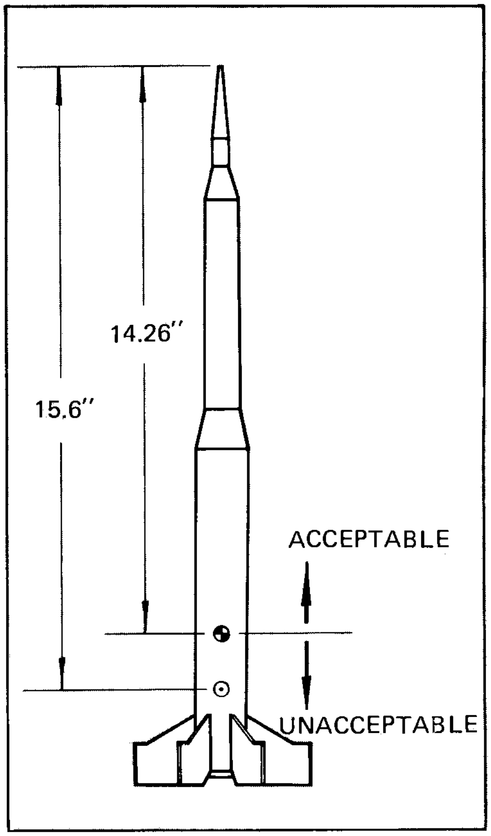


FIGURE 18

ARCON-HI EXAMPLE

A diagram of the ARCON-HI and its required dimensions is shown below.

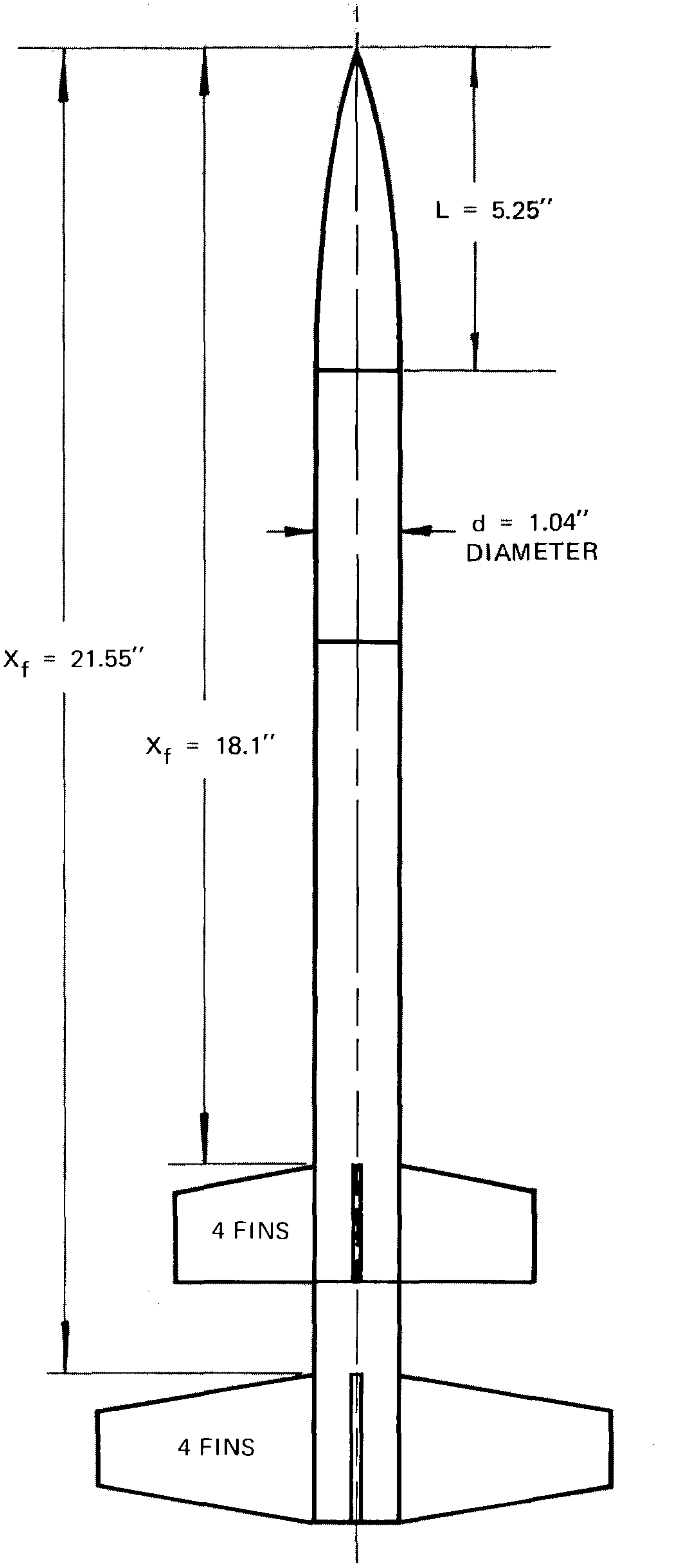


FIGURE 19

SUSTAINER FIN DIMENSIONS

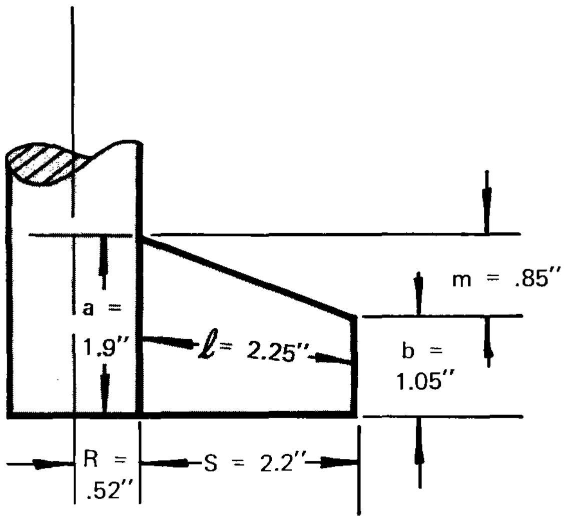


FIGURE 20

BOOSTER FIN DIMENSIONS

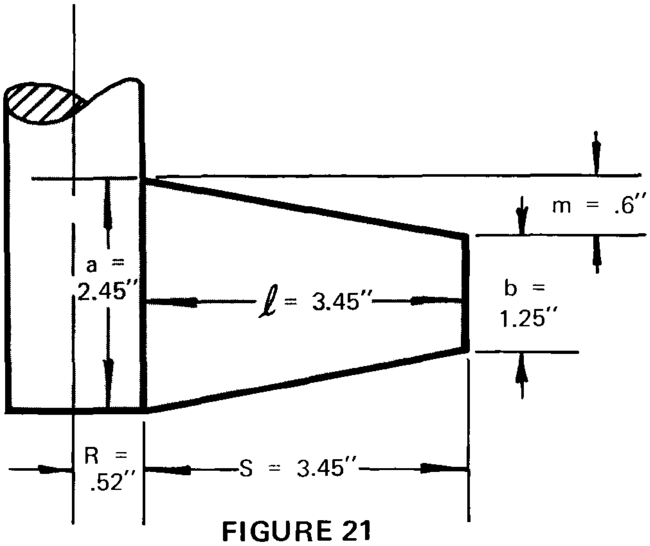


FIGURE 21

ANALYSIS OF THE TWO STAGE ARCON-HI USING THE EQUATIONS

Nose

normal force

$$(C_{N\alpha})_n = 2$$

center of pressure

$$\overline{X}_n = .466\,L$$

$$= .466\,(5.25)$$

$$\overline{X}_n = 2.45 \text{ inches}$$

Sustainer Fins

normal force on four fins

$$(C_{N\alpha})_f = \frac{4n\left(\frac{S}{d}\right)^2}{1 + \sqrt{1 + \left(\frac{2\ell}{a + b}\right)^2}}$$

$$= \frac{4\,(4)\left(\frac{2.2}{1.04}\right)^2}{1 + \sqrt{1 + \left[\frac{2\,(2.25)}{1.9 + 1.05}\right]^2}}$$

$$= \frac{16\,(2.12)^2}{1 + \sqrt{1 + \left(\frac{4.5}{2.95}\right)^2}} = \frac{16\,(4.48)}{1 + \sqrt{1 + (1.525)^2}}$$

$$= \frac{71.7}{1 + \sqrt{3.32}} = \frac{71.7}{2.82}$$

$$(C_{N\alpha})_f = 25.4$$

interference factor

$$K_{fb} = 1 + \frac{R}{S + R}$$

$$= 1 + \frac{.52}{2.2 + .52} = 1 + \frac{.52}{2.72}$$

$$K_{fb} = 1.191$$

normal force on fins in presence of body

$$(C_{N\alpha})_{fb} = K_{fb}\,(C_{N\alpha})_f$$

$$= 1.191\,(25.4)$$

$$\boxed{(C_{N\alpha})_{fb} = 30.2}$$ Sustainer

center of pressure

$$\overline{X_f} = X_f + \Delta X_f$$

$$= X_f + \frac{m(a + 2b)}{3(a + b)} + \frac{1}{6}\left(a + b - \frac{ab}{a + b}\right)$$

$$= 18.1 + \frac{.85\,[1.9 + 2\,(1.05)]}{3\,(1.9 + 1.05)}$$

$$+ \frac{1}{6}\left[1.9 + 1.05 - \frac{1.9\,(1.05)}{1.9 + 1.05}\right]$$

$$= 18.1 + \frac{.283\,(1.9 + 2.1)}{2.95} + \frac{1}{6}\left(2.95 - \frac{1.995}{2.95}\right)$$

$$= 18.1 + .0959\,(4.0) + \frac{1}{6}\,(2.95 - .68)$$

$$= 18.1 + .38 + \frac{2.27}{6} = 18.48 + .38$$

$$\boxed{\overline{X_f} = 18.86 \text{ inches}}$$ Sustainer

Booster Fins

normal force on four fins

$$(C_{N\alpha})_f = \frac{4n\left(\frac{S}{d}\right)^2}{1 + \sqrt{1 + \left(\frac{2\ell}{a + b}\right)^2}}$$

$$= \frac{4\,(4)\left(\frac{3.45}{1.04}\right)^2}{1 + \sqrt{1 + \left[\frac{2\,(3.45)}{2.45 + 1.25}\right]^2}}$$

$$= \frac{16\,(3.32)^2}{1 + \sqrt{1 + \left(\frac{6.9}{3.7}\right)^2}} = \frac{16\,(11.02)}{1 + \sqrt{1 + (1.865)^2}}$$

$$= \frac{176.4}{1 + \sqrt{4.48}} = \frac{176.4}{1 + 2.12} = \frac{176.4}{3.12}$$

$$(C_{N\alpha})_f = 56.6$$

Interference factor

$$K_{fb} = 1 + \frac{R}{S + R}$$

$$= 1 + \frac{.52}{3.45 + .52} = 1 + \frac{.52}{3.97} = 1 + .131$$

$$K_{fb} = 1.131$$

normal force on fins in presence of body

$$(C_{N\alpha})_{fb} = K_{fb}\,(C_{N\alpha})_f$$

$$= 1.131\,(56.6)$$

$$\boxed{(C_{N\alpha})_{fb} = 64.0}$$ Booster

center of pressure

$$\overline{X_f} = X_f + \Delta X_f$$

$$= X_f + \frac{m(a + 2b)}{3(a + b)} + \frac{1}{6}\left(a + b - \frac{ab}{a + b}\right)$$

$$= 21.55 + \frac{.6\,[2.45 + 2\,(1.25)]}{3\,(2.45 + 1.25)}$$

$$+ \frac{1}{6}\left[2.45 + 1.25 - \frac{2.45\,(1.25)}{2.45 + 1.25}\right]$$

$$= 21.55 + \frac{.2\,(2.45 + 2.5)}{3.7} + \frac{1}{6}\left(3.7 - \frac{3.06}{3.7}\right)$$

$$= 21.55 + .0541\,(4.95) + \frac{1}{6}\,(3.7 - .83)$$

$$= 21.55 + .27 + \frac{2.87}{6} = 21.82 + .48$$

$$\boxed{\overline{X_f} = 22.3 \text{ inches}}$$ Booster

Arcon—Hi Combination Calculations

(Booster Plus Sustainer)

total normal force

$$C_{N\alpha} = (C_{N\alpha})_n + (C_{N\alpha})_{fb} + (C_{N\alpha})_{fb}$$

$$= 2 + 30.2 + 64$$

$$\boxed{C_{N\alpha} = 96.2}$$ Booster Plus Sustainer

center of pressure of the entire rocket

$$\overline{X} = \frac{(C_{N\alpha})_n \overline{X}_n + (C_{N\alpha})_{fb} \overline{X}_f + (C_{N\alpha})_{fb} \overline{X}_f}{C_{N\alpha}}$$

$$= \frac{2\,(2.45) + 30.2\,(18.86) + 64\,(22.3)}{96.2}$$

$$= \frac{4.90 + 570 + 1428}{96.2} = \frac{2000}{96.2}$$

$$\boxed{\overline{X} = 20.8 \text{ inches}}$$ Booster Plus Sustainer

ANALYSIS OF THE ARCON-HI SUSTAINER ALONE USING THE EQUATIONS

Once the two stages have been analyzed, the sustainer can be analyzed quite simply since all the normal forces and centers of pressure have already been calculated. Essentially, the booster fin terms are dropped from the Combination Calculations.

Arcon-Hi Combination Calculations

(Sustainer Alone)

total normal force

$$C_{N\alpha} = (C_{N\alpha})_n + (C_{N\alpha})_{fb}$$

$$= 2 + 30.2$$

$$\boxed{C_{N\alpha} = 32.2}$$ Sustainer Alone

center of pressure

$$\overline{X} = \frac{(C_{N\alpha})_n \overline{X}_n + (C_{N\alpha})_{fb} \overline{X}_f}{C_{N\alpha}}$$

$$= \frac{4.9 + 570}{32.2} = \frac{575}{32.2}$$

$$\boxed{\overline{X} = 17.9 \text{ inches}}$$ Sustainer Alone

ANALYSIS OF THE TWO STAGE ARCON-HI USING THE CHARTS

Nose

(uses the equations as before)

normal force

$$\boxed{(C_{N\alpha})_n = 2}$$

center of pressure

$$\overline{X}_n = .466\, L$$

$$= .466\,(5.25'')$$

$$\boxed{\overline{X}_n = 2.45 \text{ inches}}$$

Sustainer Fins

normal force

$$\frac{S}{d} = \frac{2.2}{1.04} = 2.12\,, \quad \frac{\ell}{a + b} = \frac{2.25}{1.9 + 1.05} = \frac{2.25}{2.95} = .76$$

$(C_{N\alpha})_f = 25.5$ (from Chart 4)

interference factor

$$\frac{R}{S} = \frac{.52}{2.2} = .236$$

$K_{fb} = 1.191$ (from Chart 5)

normal force on fins in presence of body

$$(C_{N\alpha})_{fb} = K_{fb}\ (C_{N\alpha})_f$$

$$= 1.191\ (25.5)$$

$$\boxed{(C_{N\alpha})_{fb} = 30.4}$$ Sustainer

center of pressure

$$\frac{m}{a} = \frac{.85}{1.9} = .45\,, \quad \frac{b}{a} = \frac{1.05}{1.9} = .55$$

$\frac{\Delta X_f}{a} = .405$ (from Chart 6)

$$\overline{X}_f = X_f + \left(\frac{\Delta X_f}{a}\right) a$$

$$= 18.1 + .405\,(1.9) = 18.1 + .76$$

$$\boxed{\overline{X}_f = 18.86 \text{ inches}}$$ Sustainer

Booster Fins

normal force

$$\frac{S}{d} = \frac{3.45}{1.04} = 3.32 \;,\; \frac{\ell}{a+b} = \frac{3.45}{2.45+1.25} = \frac{3.45}{3.7} = .93$$

$(C_{N\alpha})_f = 57.$ (from Chart 4)

interference factor

$$\frac{R}{S} = \frac{.52}{3.45} = .151$$

$K_{fb} = 1.131$ (from Chart 5)

normal force on fins in presence of body

$$(C_{N\alpha})_{fb} = K_{fb}\ (C_{N\alpha})_f$$

$$= 1.131\ (57)$$

$(C_{N\alpha})_{fb} = 64.4$ Booster

center of pressure

$$\frac{m}{a} = \frac{.6}{2.45} = .245 \;,\; \frac{b}{a} = \frac{1.25}{2.45} = .51$$

$$\frac{\Delta X_f}{a} = .305 \text{ (from Chart 6)}$$

$$\overline{X}_f = X_f + \left(\frac{\Delta X_f}{a}\right) a$$

$$= 21.55 + .305\ (2.45) = 21.55 + .75$$

$\overline{X}_f = 22.3$ inches Booster

Arcon-Hi Combination Calculations

(Booster Plus Sustainer)
uses equations as before

total normal force

$$C_{N\alpha} = (C_{N\alpha})_n + (C_{N\alpha})_{fb} + (C_{N\alpha})_{fb}$$

$$= 2 + 30.4 + 64.4$$

$C_{N\alpha} = 96.8$ Booster Plus Sustainer

center of pressure of the entire rocket

$$\overline{X} = \frac{(C_{N\alpha})_n \overline{X}_n + (C_{N\alpha})_{fb} \overline{X}_f + (C_{N\alpha})_{fb} \overline{X}_f}{C_{N\alpha}}$$

$$= \frac{2\ (2.45) + 30.4\ (18.86) + 64.4\ (22.3)}{96.8}$$

$$= \frac{4.9 + 574 + 1437}{96.8} = \frac{2016}{96.8}$$

$\overline{X} = 20.8$ inches Booster Plus Sustainer

ANALYSIS OF THE ARCON-HI SUSTAINER USING THE CHARTS

The procedure simply involves dropping the booster terms from the Combination Calculations.

Arcon—Hi Combination Calculations

(Sustainer Alone)

total normal force

$$C_{N\alpha} = (C_{N\alpha})_n + (C_{N\alpha})_{fb}$$

$$= 2 + 30.4$$

$C_{N\alpha} = 32.4$ Sustainer Alone

center of pressure of the entire rocket

$$\overline{X} = \frac{(C_{N\alpha})_n \overline{X}_n + (C_{N\alpha})_{fb} \overline{X}_f}{C_{N\alpha}}$$

$$= \frac{4.9 + 574}{32.4} = \frac{579}{32.4}$$

$\overline{X} = 17.9$ inches Sustainer Alone

ARCON-HI STABILITY CHECK

The C.P. and C.G. relations for both flight configurations (booster plus sustainer and sustainer alone) must be established since the rocket must be stable for both conditions. One caliber stability will in both cases be based on the body diameter (D).

$$D = 1.04''$$

At lift-off the C.G. can be no farther back than

$$X_{CG} = \overline{X} - D$$

$$= 20.8 - 1.04$$

$X_{CG} = 19.76$ inches from the nose

For stable second-stage flight, the C.G. can be no farther back than

$$X_{CG} = \overline{X} - D$$

$$= 17.9 - 1.04$$

$X_{CG} = 16.86$ inches from the nose

The location of acceptable balance points for both configurations is shown in the illustration on the next page.

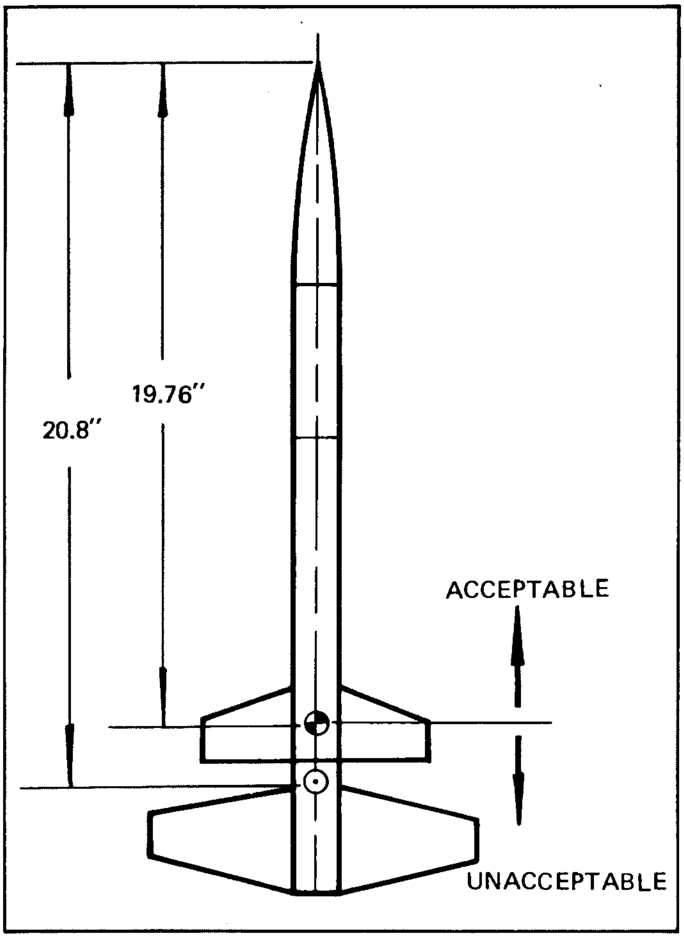
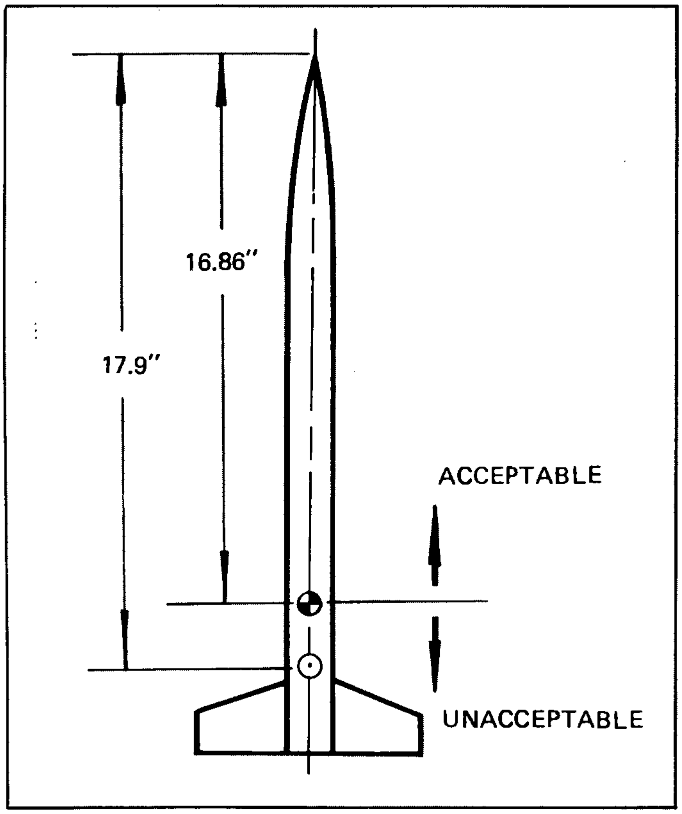


FIGURE 22



9. APPENDIX A—ESTIMATING THE CENTER OF GRAVITY OF MODEL ROCKET DESIGN

Until a model rocket design is completely built, painted and ready to fly, its center of gravity (C.G.) can only be estimated. The way a model is glued, sanded, and finished can strongly affect its final C.G. location. Because of this, a new design should always be balance tested before it is flown. If the static margin isn't adequate, the C.G. location can then be changed and the rocket re-balanced by the proper addition or removal of weight. However, if the C.G. is estimated carefully and the construction and finishing of the rocket is well done, not much re-balancing will be needed.

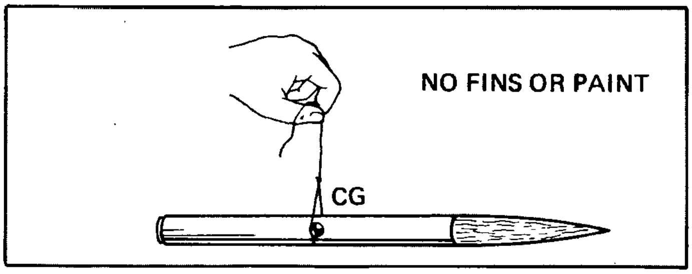
There are two basic techniques that can be used to estimate the C.G. location of a new rocket design. Both of them concentrate on the body of the rocket where most of the rocket's weight is concentrated. Remember, when you are finding the weight and C.G. location of a rocket, you must always include the largest engine expected to be used for a flight.

BALANCE TEST TECHNIQUE

When you are designing a model rocket, you will usually have on hand all the parts of the model you are going to build. Since the body design must be known before the fins can be designed, the C.G. location of the body is easily found by a simple balance test. The steps for doing this are similar to the normal procedure for building a model.

1. Decide what the design of the body will be.
2. Fit the body parts together or actually build the body.
3. Insert the engine(s) and parachute.
4. Determine C.G. location of the body by the string balance test.

FIGURE 23



If the rocket design fits any of the following descriptions, then the fins and painting will not change the C.G. location appreciably.

1. The overall length of the body is greather than twelve times its largest diameter.
2. The design has more than one engine (two or three stages or clusters).

3. The design has a payload that weighs at least as much as an engine.

Thus, if any of the above conditions is satisfied, the C.G. location of the design will be at the point determined by the balance test alone.

If none of the above conditions are satisfied, the effect of the fins can be estimated by assuming that the fins will weigh 5% of the total body weight and that the fin C.G. will be at the base of the rocket. Then the overall C.G. location can be estimated using the moment balance equation.

$$X_{CG} = \frac{(X_{CG})_B + .05L}{1.05}$$

where:

X_{CG} = C.G. location of body plus the fins measured from the nose tip.

$(X_{CG})_B$ = C.G. location, measured from the nose tip, of the body alone determined by the balance test.

L = Total length of the body.

For example, if the balance test showed that the C.G. of the body alone was located at 6.3 inches from the nose; and the body was 10 inches long over-all; then the estimated C.G. location would be,

$$X_{CG} = \frac{6.3 + .05(10)}{1.05} = \frac{6.3 + .5}{1.05} = \frac{6.8}{1.05}$$

X_{CG} = 6.48 inches from the nose.

As expected, the C.G. has shifted somewhat to the rear. Alternatively, the same answer would be obtained by adding 5% of the total body weight in the form of modeling clay to the base of the rocket and simply finding the new balance point. This is, after all, exactly what the above equation is physically simulating.

CALCULATION TECHNIQUE

Of course, it is not always possible to fit together or build your body design. If this is the case, you must rely on the theoretical calculations to estimate the body C.G. location. An outline of how to do this is given below.

1. Determine the weight of each individual component of the body design (nose cone, body tube, engine, etc.) either by weighing it or by using the net weight given in the CENTURI catalog. Small light parts such as launch lugs and detailing don't have to be considered.

2. Determine the C.G. location of each individual component. Some reasonable approximations of the C.G. locations of different components are given below.

   a. Cylindrical-shaped components, such as body tubes, balsa plugs, engine mounts, thrust rings, tubing couplers, rolled streamers, and engines have their C.G.'s at their midpoints.

   b. Nose cones and reduction fittings made of solid balsa will have their C.G.'s at about 2/3 their total length from the narrow end, including the parts that fit inside the body tube.

   c. The parachute, shock cord, and lines can be considered as a single package which will have its C.G. at the middle of its length when packed into the body tube.

3. Make an accurate full scale or scaled drawing of the body design that shows the placement of each component. Mark the appropriate C.G. location on each piece.

4. Measure the distance between the nose tip and the C.G. location of each component. If the drawing is scaled, make sure you take this into account when you measure.

5. Add the weights of the individual components to get the total weight of the body. In equation form,

$$W_B = W_1 + W_2 + W_3 + W_4 + \ldots\ldots\ldots$$

   where:

   W_B is the total weight of the body.

   Each W with a numbered subscript (W_1, W_2, W_3, W_4) represents the weight of an individual component.

   The dots (. . . .) at the end indicate that the numbers can go as high as is necessary, depending on the total number of different components.

6. Multiply the weight of each individual component by the distance between its C.G. and the nose tip.

7. Add together all the numbers resulting from Step 6.

8. Divide the result of the addition in Step 7 by the total body weight from Step 5. The result of this division is the body C.G. location, measured from the nose tip.

The last three Steps 6, 7, and 8 can be represented by the equation:

$$(X_{CB})_B = \frac{W_1(X_{CG})_1 + W_2(X_{CG})_2 + W_3(X_{CG})_3 + \ldots}{W_B}$$

where:

$(X_{CG})_B$ is the body C.G. location, measured from the nose tip.

Each X_{CG} with a numbered subscript $(X_{CG})_1$, $(X_{CG})_2$, $(X_{CG})_3$ . . . . represents the distance between the nose tip and the C.G. of an individual component.

Notice that the above equation is a moment balance, just like the combination calculation equation used in determining the C.P.

Once the body C.G. location has been calculated, the effect of the fins can be estimated by the same method given in the Balance Test Technique section.

10. APPENDIX B — THEORY OF MOMENTS

The tendency of a force to rotate a body about a certain point is known as the moment of the force about this point. A mathematical formula for this tendency can be written as follows:

(Moment) = (Force) X (Moment Arm)

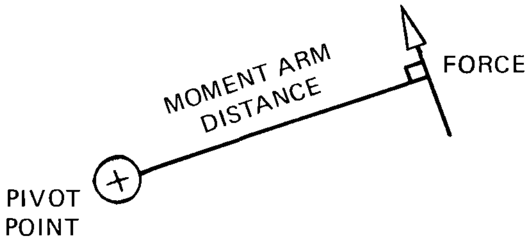


FIGURE 24

It can easily be seen that this formula physically represents the rotation tendency of interest. If the force is made larger, the moment and the tendency to rotate become proportionally larger. Similarly, if the moment arm (or lever arm) is increased, the same force will produce a corresponding larger moment.

Notice that we used an arrow to represent the force. It is very useful to do this on drawings since the arrow can represent the properties of magnitude and direction which are associated with "forces".

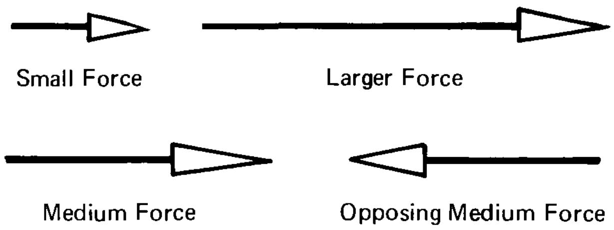


FIGURE 25

If the first three forces were considered to be acting in a positive (+) direction then the opposing force is acting in a negative (–) direction.

When the total normal force at a rocket's center of pressure is represented as an arrow, we can immediately see which direction the rocket would tend to rotate.

In the case of a stable rocket, the normal force produces a moment about the pivot point (the C.G.) which tends to bring the rocket back to zero degree angle-of-attack. Simultaneously, if the C.P. is ahead of the C.G., using an arrow to represent the normal force should help visualizing that the rocket has a tendency to keep increasing its angle-of-attack resulting in an unstable flight condition.

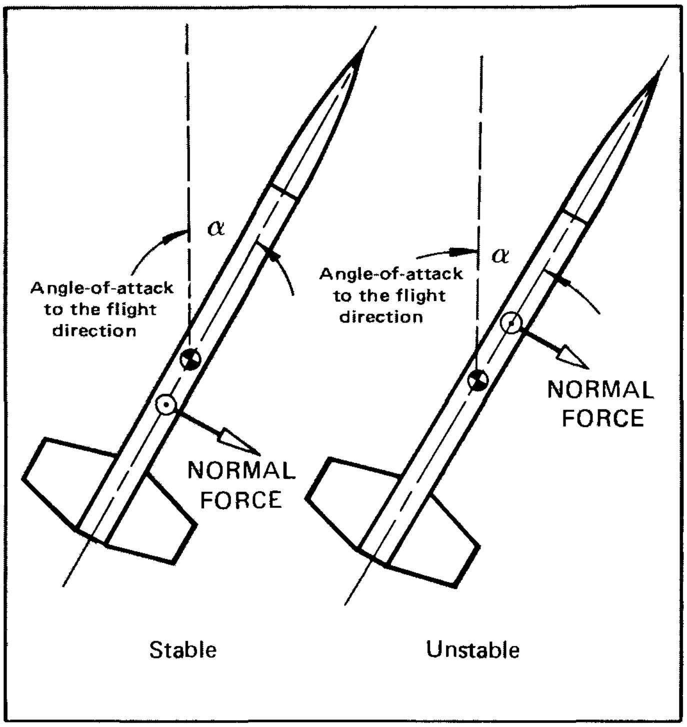


FIGURE 26

Physical quantities that possess both magnitude and a specified direction are referred to in mathematics as vectors. Some specific vectors model rocketeers will come across in their studies are velocity, acceleration, thrust, and aerodynamic drag.

Vector forces are common in everyday living; so are moments. Every time you open a door you are applying a force in a specific direction which produces a moment.

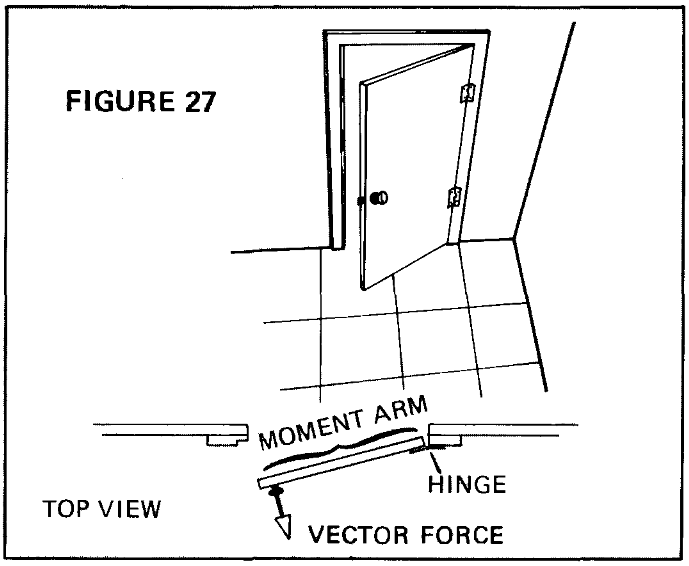


FIGURE 27

If you pull harder, the moment is larger and the door opens faster. We may ask ourselves why aren't door knobs placed as shown below?

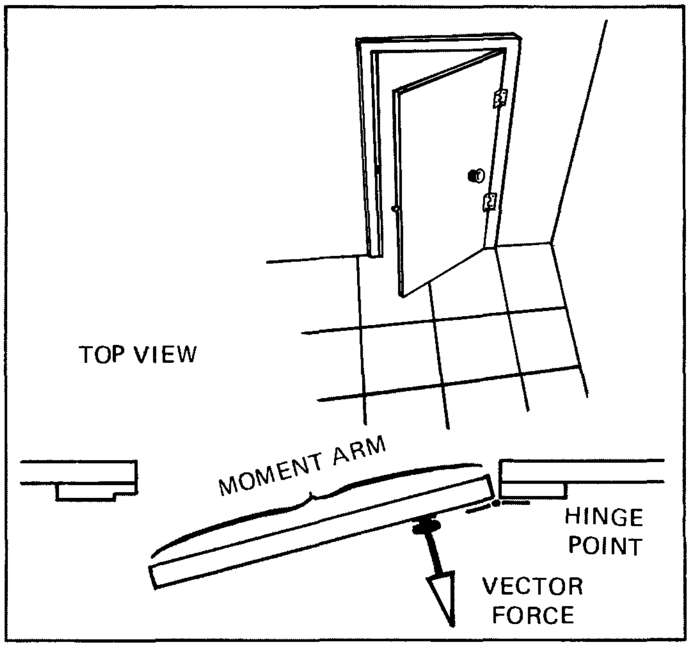


FIGURE 28

That's right! To get the door to respond in the usual manner you have to reproduce the usual moment. Since the moment armis so short, you have to greatly increase the applied force in order to open the door. If you were having a strength contest using a door to push against as shown below, which of the two positions would you choose?

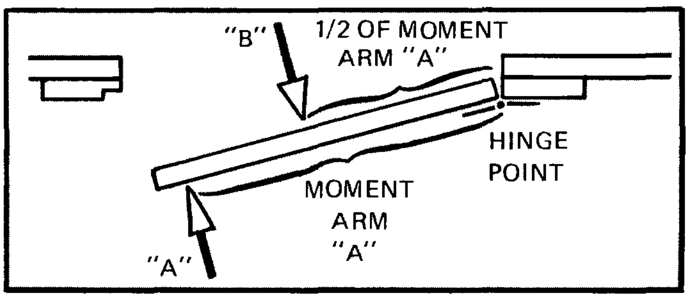


FIGURE 29

Intuitively, most everyone would pick position "A". If you think about it a while you'll mathematically understand why "A" has an advantage. The moment about the hinge point produced by "B" is his total pushing force times the moment arm. To match that moment, "A" only has to push half as hard as "B", since "A"'s moment arm is twice as long. If "A" pushes anything slightly over half as hard, he will cause "B" to lose.

You may want to experimentally verify the theory of moments using your dad's help. It is an excellent way to also verify if he is still twice as strong as you are.

We mentioned earlier that continual corrections to disturbances are being applied when riding a bicycle or steering a car. In both cases these corrections are also due to moments produced by vector forces.

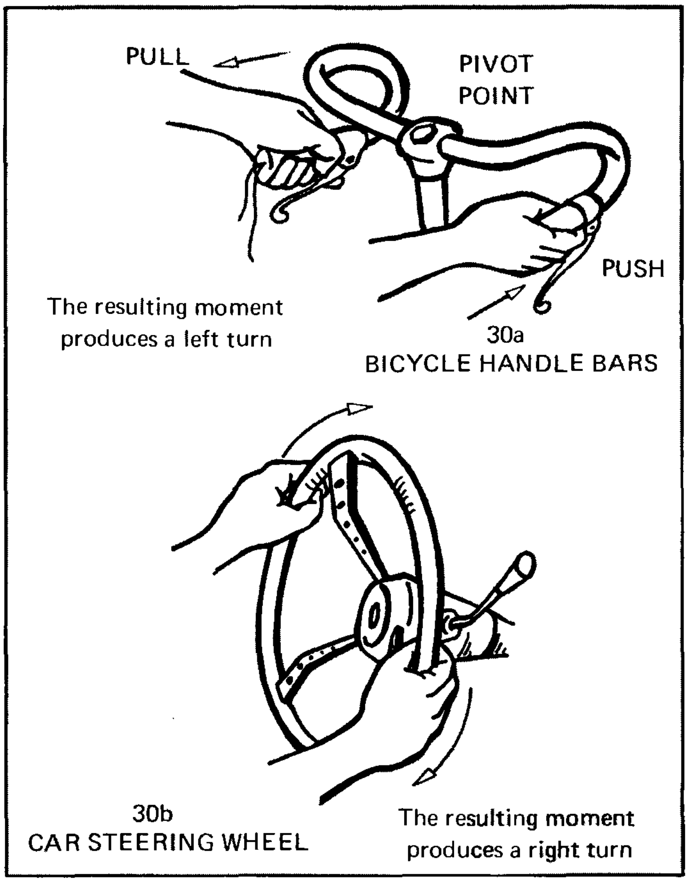


FIGURE 30

11. APPENDIX C—RESOLUTION OF FORCES

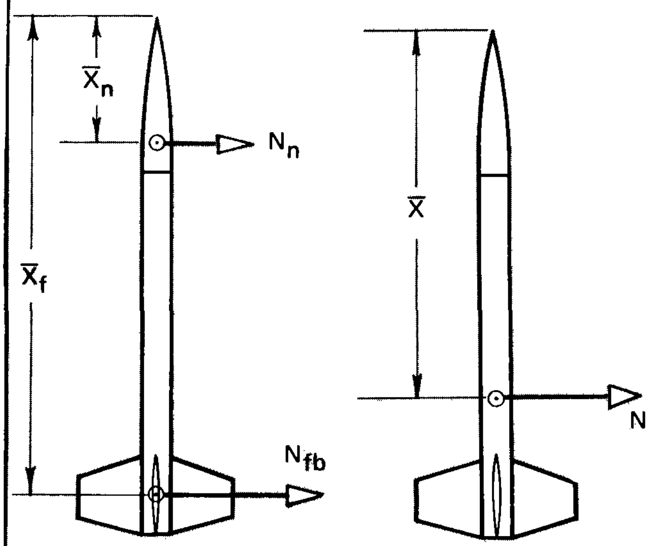
Scientists and engineers gain understanding of complex problems by simplifying them as much as possible. It would be very difficult and time consuming for us to draw general conclusions concerning stability if we could only work with the actual pressure force distribution over the rocket as shown previously in Figure 3. Knowing this distribution meant nothing until it was discovered that by simplifying the model into separate regions, general equations could be developed that would give the total normal force on these regions.

However, even knowing the force on each region, in itself, doesn't help in deciding if a rocket will be stable or not. Only by completing one more simplification does the stability question reduce to one that can be understood easily. This last reduction in complexity involves replacing all the forces acting separately on each region by a single force which would physically cause the same effect on the rocket in free flight. This is called resolving the forces. In effect what has been termed the "total normal force" throughout the report doesn't really exist. Applied to the rocket though, this fictional force would produce the exact same effect on the rocket's motion as the actual distributed air pressure forces.

In other words, the single force has a magnitude equal to the sum of all the actual distributed forces and most important it produces the same moment about the pivot point that the actual distributed forces produce.

12. APPENDIX D ------- WHY $C_{N\alpha}$ CAN BE USED TO REPRESENT THE TOTAL NORMAL FORCE (N)

The following derivation, using the principle of resolution of forces, demonstrates why it is mathematically acceptable to replace the normal forces by their associated dimensionless coefficients ($C_{N\alpha}$'s) in the moment balance equations.



Normal forces on nose and fins

Total normal force

FIGURE 31

The single force must have the same value as the total of the two separate forces

$$N = N_n + N_{fb}$$

The total moment about the reference point due to the two actual normal forces is

$$M_1 = N_n\overline{X}_n + N_{fb}\overline{X}_f$$

and the moment about the reference point due to the single replacement force will be

$$M_2 = N\overline{X} = (N_n + N_{fb})\overline{X}$$

where $\overline{X}$ is the presently unknown location for the replacement force.

Next, by writing a balance equation we make the moment produced by the replacement force identical to the actual total moment.

$$M_2 = M_1$$
$$N\overline{X} = N_n\overline{X}_n + N_{fb}\overline{X}_f$$

Dividing both sides by the single force N, we obtain an equation for the unknown location of the single replacement force

$$\frac{N\overline{X}}{N} = \overline{X} = \frac{N_n\overline{X}_n + N_{fb}\overline{X}_f}{N}$$

or

$$\boxed{\overline{X} = \frac{N_n\overline{X}_n + N_{fb}\overline{X}_f}{N + N_{fb}}}$$

Now we write the exact equation for each normal force

$$N_n = (C_{N\alpha})_n \tfrac{1}{2}\rho V^2 \alpha A_r$$

$$N_{fb} = (C_{N\alpha})_{fb} \tfrac{1}{2}\rho V^2 \alpha A_r$$

Substitution of the exact values into the $\overline{X}$ equation we get

$$\overline{X} = \frac{\left[(C_{N\alpha})_n \tfrac{1}{2}\rho V^2 \alpha A_r\right]\overline{X}_n + \left[(C_{N\alpha})_{fb} \tfrac{1}{2}\rho V^2 \alpha A_r\right]\overline{X}_f}{(C_{N\alpha})_n \tfrac{1}{2}\rho V^2 \alpha A_r + (C_{N\alpha})_{fb} \tfrac{1}{2}\rho V^2 \alpha A_r}$$

notice that the $\tfrac{1}{2}\rho V^2 \alpha A_r$ can be withdrawn from the equations. This gives

$$\overline{X} = \frac{\left[(C_{N\alpha})_n \overline{X}_n + (C_{N\alpha})_{fb}\overline{X}_f\right] \tfrac{1}{2}\rho V^2 \alpha A_r}{\left[(C_{N\alpha})_n + (C_{N\alpha})_{fb}\right] \tfrac{1}{2}\rho V^2 \alpha A_r}$$

and it is easy to see that these terms cancel, leaving

$$\overline{X} = \frac{(C_{N\alpha})_n \overline{X}_n + (C_{N\alpha})_{fb}\overline{X}_f}{(C_{N\alpha})_n + (C_{N\alpha})_{fb}}$$

which is the equation we've been using throughout the report to find the overall center of pressure.

Thus, even the $C_{N\alpha}$ is just one of the factors affecting the normal force acting on the rocket, it is the only factor which varies for each region.

13. APPENDIX E -- ROCKET FLEXIBILITY

An interesting point can be made at this time regarding cancelling the angle-of-attack from the previous equation since they were all identical. Assumption 6 specified that the rocket must be an axially symmetric rigid body. If a model rocket is not a rigid body and can bend, various parts of it could have different angles-of-attack.

The angle-of-attack of each region would have to be accounted for in the overall center-of-pressure equation. If the rocket was flexible enough to bend then the distributed normal pressure forces could deflect the rocket so that the fins are at say ½ degree angle-of-attack and the nose would be at say 2 degrees angle-of-attack as shown.

This kind of problem exists with all real missiles to some degree and is carefully accounted for by the engineers doing center-of-pressure calculations.

One of our country's earliest Satellite launch vehicles, the Vanguard lacked adequate lengthwise stiffness, (it was very long and slender) and that section of the rocket containing the gyro control sensed angles-of-attack which had nothing to do with where the nose of the vehicle was pointed. Needless to say, it caused problems. Fortunately, the model rocket kits of today are generally more than stiff enough and the assumption of a rigid body is quite realistic.

Watch out though if you design a long rocket that has a break in the middle. If it is not a good fit and has some wiggle to it, the nose could quite easily get to a higher angle-of-attack than the fins. This would create proportionally larger normal force ahead of the C.G.; possibly more than the fins produce behind the C.G. and could bring the new resultant center of pressure forward —— ahead of the C.G. This example does not actually represent a flexible body which bends gradually over its entire length, but instead is really two rigid bodies connected by a sloppy mechanical joint. Either way, this still violates assumption 6 and, as such, even careful C.P. analysis may not produce useable results.

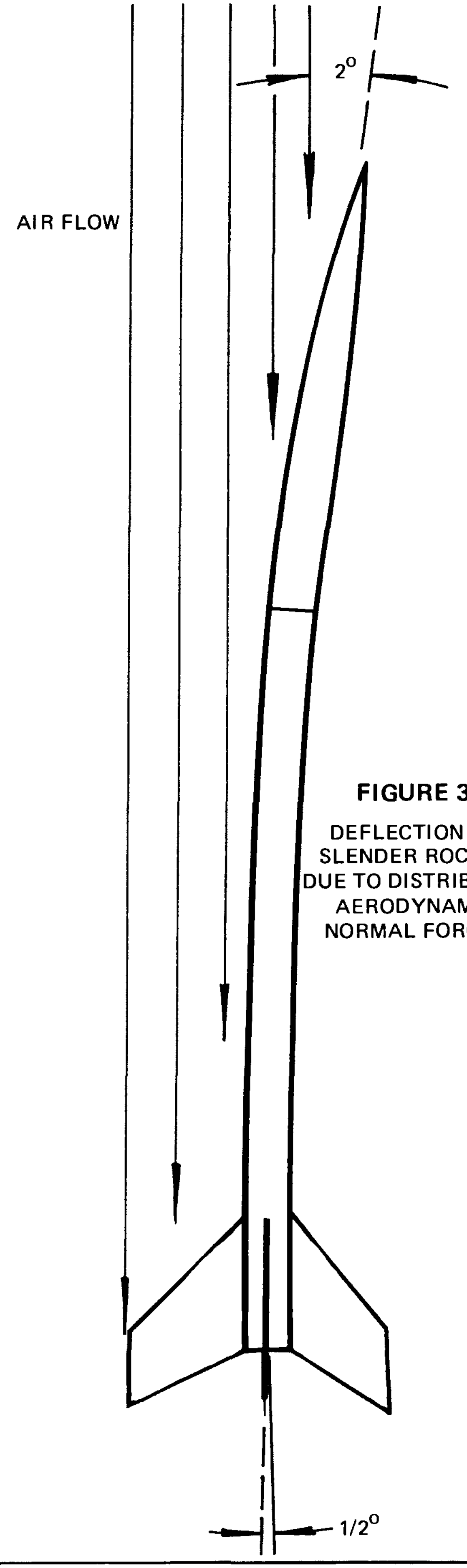


FIGURE 32

DEFLECTION OF SLENDER ROCKET DUE TO DISTRIBUTED AERODYNAMIC NORMAL FORCES

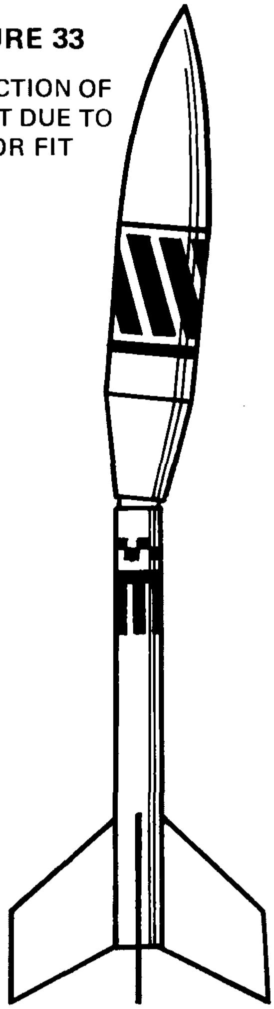


FIGURE 33

DEFLECTION OF ROCKET DUE TO POOR FIT

ABOUT THE AUTHOR

JIM BARROWMAN

JIM BARROWMAN is presently employed by NASA's Goddard Space Flight Center in Greenbelt, Maryland as an Aerospace Engineer in Fluid and Flight Dynamics. Jim was born in Toledo, Ohio 25 years ago and graduated from the University of Cincinnati in 1965 with a Bachelor of Science degree in Aerospace Engineering. Jim, together with his wife Judy and their two year old daughter Julie Ann, presently reside in Hyattsville, Maryland where he continues graduate level studies at nearby Catholic University of America.

He has been employed by NASA since 1961, and worked as a co-op student trainee during the first four years. Here he performed magnetometer data reduction for the Vanguard III Satellite, was a member of a Mars Atmospheric Entry Capsule design team, assisted in the thermal design of the IMP (Interplanetary Monitoring Probe) Spacecraft, performed dynamic motion studies of the Aerobee 150A Sounding Rocket, and wrote a computer program for his aerodynamic analysis of the Tomahawk, Nike-Tomahawk, and Black Brant IIIB Sounding Rockets.

Jim's interest in Model Rocketry dates back to 1964. He enjoys working with young people, and in addition to occasional lectures to Junior and Senior High School groups on aerospace careers, he has become the senior advisor for the NARHAMS Section of the National Association of Rocketry. The method Jim developed for calculating the exact center of pressure of a model rocket earned him a First Place Senior Research and Development Award at NARAM-8 in August, 1966. He is also an active NAR Trustee and has been appointed Chairman of the NAR Publications Committee and Contest Director for NARAM-10. Jim's obviously few leisure moments are spent sailing and experimenting with his favorite model rocket - - - the boost glider.

TECHNICAL INFORMATION REPORTS

STABILITY OF A MODEL ROCKET IN FLIGHT TIR-30

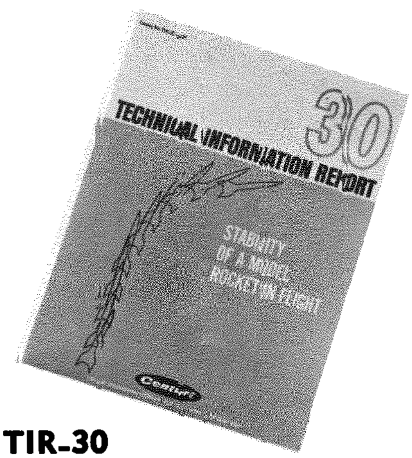
BY JAMES BARROWMAN

Highly recommended for a non-theoretical explanation of stability. TIR-30 does not delve into any of the mathematics of calculating the center of pressure covered here in TIR-33 and, as such, provides an easy-to-follow introduction to the subject. The report provides valuable information to those who want to gain insight and a true understanding of basic stability concepts.

Soon after being introduced to model rocketry, most rocketeers hear the statement that a stable rocket flight requires that the center of pressure must lie behind the center of gravity.

What is center of pressure? Why is the rocket's balance point called center of gravity? What does the word "stability" really mean? Are there any simple tests which tell you whether or not a new rocket design will be "stable"? How come rockets arc over and head into the wind (weathercock) during thrusting and coasting instead of being blown along with the wind as a feather or piece of paper would?

We think that these and other important questions are carefully and clearly answered in CENTURI's TIR-30 with the assistance of a total of 42 illustrations. In addition, a section has also been included on how the amount of stability can be adjusted to improve altitude performance.



TIR-30
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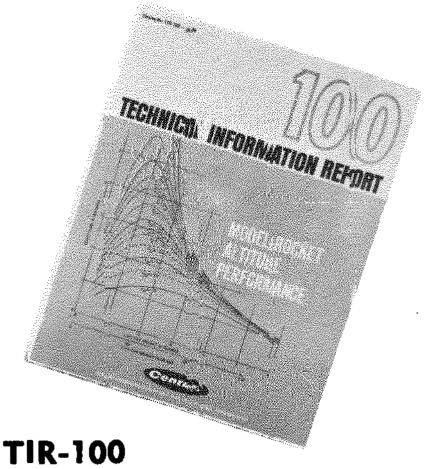
MODEL ROCKET ALTITUDE PERFORMANCE TIR-100

BY DOUGLAS MALEWICKI

Presents easy-to-use graphs for accurately predicting the peak altitudes which can be reached by single-stage rockets using ¼A thru F type engines. Also included are graphs for selecting the best delay time to use. No mathematical calculations, whatever, are involved in finding altitudes or engine delay times. These graphs, along with the numerous discussion sections of this report, should be most useful in helping the rocketeer towards a real understanding of how engine power, rocket weight, and aerodynamic drag on various nose and body shapes are interrelated in their affects on performance.

All the altitude data in this report is based entirely on Centuri's latest model rocket engines. The National Association of Rocketry (NAR), the Federation Aeronautique Internationale (FAI), and the United States Model Rocket Manufacturers Association have all recently adopted the Metric System of measurement. As a result, Centuri model rocket engines were redesigned to give the maximum total impulse allowed in each new Metric category. These modifications mean that the new engines have slightly different average thrust levels and thrust duration characteristics than the old engines and TIR-100 properly reflects these changes.

Also note that altitude performance graphs for the new "C" type engines with time delays are included. These new engines have 50 per cent more total impulse than the old C.8-0 booster engines and twice the total impulse of the old "B" type engines.



TIR-100
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