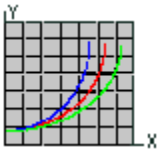
Richard Nakka's *Experimental Rocketry* Web Site



Solid Rocket Motor Theory -- Introduction

Introduction

The primary goal of the *Solid Rocket Motor Theory* Web Pages is to present the theoretical basis for the functioning of a solid propellant rocket motor. Emphasis is placed on the theory as it applies to small (relatively speaking) amateur motors, which are typically of lower performance and efficiency than their "professional" counterparts. In certain regard, the standard "textbook" methods must be modified to take these factors into account.

The secondary goal is to present the fundamental "tools" that may be used in the design of amateur rocket motors. This topic will be covered in a later Web Page.

I will attempt to follow a logical path in this presentation, with one topic leading into the next topic. Starting with the **Basic Assumptions** that must be made in order to "simplify" the highly complex nature of the functioning of a rocket motor. The **Propellant** is then discussed, although mainly with regard to certain aspects that have direct application to rocket motor theory. For example, propellants are considered to consist of a fuel/binder and oxidizer, without regard to specific formulations. The shape that the propellant is formed into (the *grain*) has a direct and important bearing on the motor's overall performance characteristics.

As the fundamental operation of the rocket motor requires that the propellant be burned, the topic that follows deals with the **Combustion** process. In other words, the conversion of the propellant grain to high temperature gases and condensed particles (smoke). This combustion must occur in a manner that is suitable to obtain the desired operating requirements -- a certain thrust profile over a certain burn time, while operating within certain physical limits with regard to chamber pressure and temperature. Thus, burn rate, combustion temperature, and products of combustion all play a crucial role in establishing a rocket motor's performance.

Almost certainly the most critical component of a solid rocket motor is the nozzle. The nozzle can "make or break" a rocket motor, most literally. But what exactly does a nozzle do, and how? What is the signficance of the convergent and divergent profile? These questions are addressed in the section on **Nozzle Theory**.

The expulsion of the exhaust products through the nozzle at high velocity produces thrust, the "power" of a rocket motor. Thrust may be measured through fairly simple means, but how does one predict what the theoretical thrust will be for a given motor design? The sections that follow discuss the means to calculate thrust, as well as **Total Impulse** and **Specific Impulse**. The latter two parameters are the "yardsticks" to measure the useful "propulsive power" of a motor, and the "worth" of a specific propellant in this regard.

Anyone who has familiarity with rocket motors knows that they operate under high **Pressure**. It is this chamber pressure, produced by the combustion of propellant, that forces the exhaust out of the motor via the nozzle. Controlling this pressure is the key to more successful, and safer, rocket motor design and operation. What parameters determine the pressure, which can be looked upon quite realistically as a "controlled explosion" within the combustion chamber?

The final topic dealt with in the theory of rocket motors is the "**Corrections**" that must be considered in order to bridge the theoretical predictions to the true results that will be obtained in an "**Actual**" rocket motor. These corrections are a direct result of the topic first dealt with, that is, the simplified assumptions that make such an analysis at all possible.

The final two topics covered deal with one of a number of software tools that greatly eases the most difficult and laborious procedure in analysing the operation of a rocket motor -- the combustion process. This software exists in various forms, such as PROPEP, but also referred to as GUIPEP, NEWPEP, PEP (which are all essentially the same program), as well as CET. The acronyms are as follows: PEP = **P**ropellent **E**valuation **P**rogram; CET = **C**hemical **E**quilibrium with **T**ransport Properties. The meaning of the various (often cryptic) terms that are printed in the output file is explained, as well as how these results are derived. Additionally, a brief writeup describing the basic workings of the program is presented.

These programs have certain shortcomings that may not be of importance to large scale "professional" rocket motors and propellants, but may certainly have a significant impact on the predicted performance of amateur propellants. This topic is dealt with in the final section of the *Solid Rocket Motor Theory* Web Pages.

Next -- Basic Assumptions

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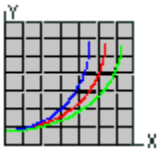

Solid Rocket Motor Theory -- Basic Assumptions

Basic Assumptions

The various physical and chemical processes that occur in an actual rocket motor during operation are highly complex. These processes include the complex chemical reactions that occur during combustion; the manner in which "consumption" of the propellant grain occurs during burning; the behaviour of the flow of exhaust gases as they form at the burning surface, travel through the chamber, and exit through the nozzle; the interaction between the exhaust gases and condensed particles (smoke).
The theoretical analysis of a solid rocket motor necessitates certain simplifications, that is, the assumption is of an *ideal rocket motor*. An ideal rocket motor assumes the following:

- The propellant combustion is complete and does not vary from that assumed by the combustion equation.
- The combustion products obey the *perfect gas law*.
- There is no friction impeding the flow of exhaust products.
- The combustion and flow in the motor and nozzle is *adiabatic*, that is, no heat loss occurs to the surroundings.
- Unless noted otherwise, *steady-state* conditions exist during operation of the motor. This means that the conditions or processes that occur do not change with time (for a given geometric conditions) during burning.
- Expansion of the working fluid (exhaust products) occurs in a uniform manner without shock or discontinuites.
- Flow through the nozzle is one-dimensional and non-rotational.
- The flow velocity, pressure, and density is uniform across any cross-section normal to the nozzle axis.
- Chemical equilibrium is established in the combustion chamber and does not shift during flow through the nozzle. This is known as "frozen equilibrium" conditions.
- Burning of the propellant grain always progresses *normal* (perpendicular)

to the burning surface, and occurs in a uniform manner over the entire surface area exposed to combustion.

Any further assumptions that may be required are stated as necessary in the following analyses.
Although it seems like a lot of simplifying assumptions must be made, in fact, these are all reasonable and can be expected to reflect the actual behaviour of the rocket motor fairly closely.

Next --Propellant Grain

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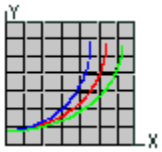

Solid Rocket Motor Theory -- Propellant Grain

Propellant Grain

The propellant utilized in amateur experimental rocket motors may be simple in composition, being comprised of two main constituents -- fuel and an oxidizer. Such is the case with the "sugar" based propellants. Experimental *composite* propellants, on the other hand, may have a composition that is fairly complex, and may contain oxidizer of various mesh sizes, polymer binder, and even metals such as aluminum or magnesium. Curing agents, phase stabilizers, and solvents may be other additives included in small percentages.
For any propellant, additives may control the burnrate, either to accelerate or to slow the rate. An opacifier may be added to absorb heat that may otherwise be transmitted through a translucent grain resulting in unpredictable burning.
Regardless of the composition, however, all propellants are processed into a similar basic **geometric form**, referred to as a propellant *grain*. As a rule, propellant grains are cylindrical in shape to fit neatly into a rocket motor in order to maximize *volumetric efficiency*. The grain may consist of a single cylindrical *segment* (Figure 1), or may contain many segments. Usually, a central *core* that extends the full length of the grain is introduced, in order to increase the propellant surface area initially exposed to combustion.

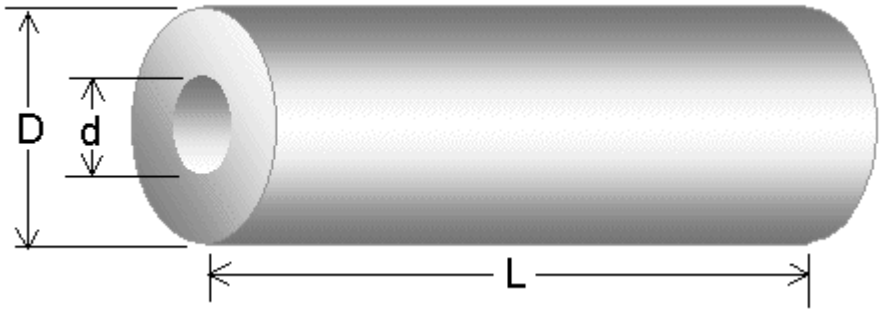

Figure 1 -- Hollow cylindrical grain

The core may have a wide variety of cross-sections such as circular, star, cross, dog-bone, wagon-wheel, etc., however, for amateur motors, the most common shape is circular. The core shape has a profound influence on the shape of the thrust-time profile, as shown in Figure 2.

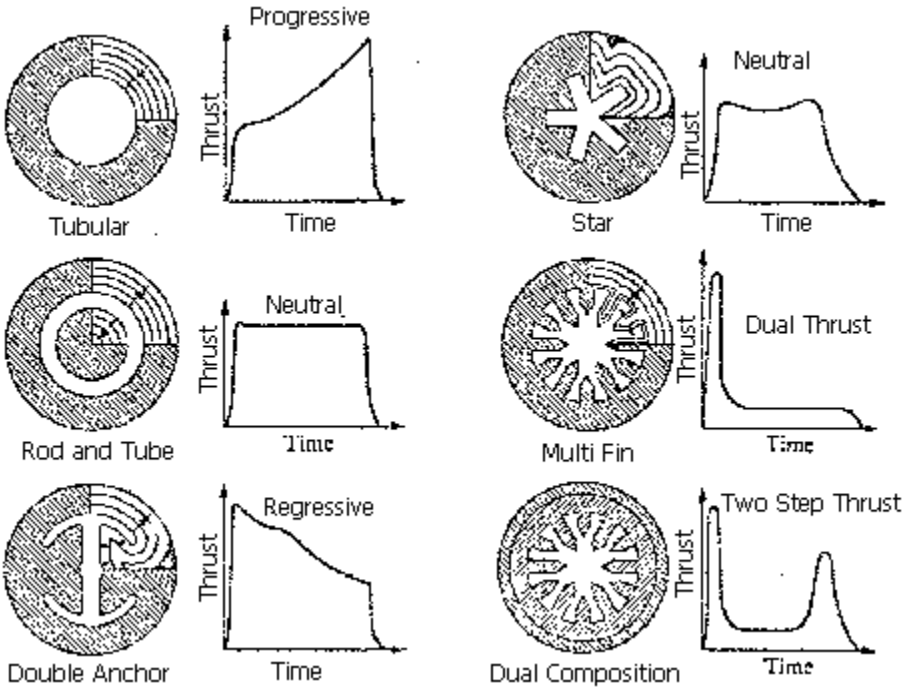

Figure 2 -- Core shapes and influence on thrust curve
Click for more...

How does the core shape influence the thrust-time curve? The thrust (and chamber pressure) that a rocket motor generates is proportional to the burning area at any particular instant in time. This is referred to as the *instantaneous burning area*. The burning surface at any point recedes in the direction normal (perpendicular) to the surface at that point, the result being a relationship between burning surface and web distance burned that depends almost entirely on the *grain initial shape* and restricted (inhibited) boundaries. This important concept is illustrated in Figure 3, where the contour lines represent the core shape at successive moments in time during the burn. Notice that the shape of the thrust-time curve changes, with the vertical lines corresponding the the same successive moments during the burn. As can be seen , the star grain provides an approximately *neutral* burn, as the surface area remains fairly constant throughout the burn duration. A neutral burn is usually desirable

because it provides for greater efficiency in delivery of total impulse, as a nozzle operates most efficiently at a constant chamber pressure.

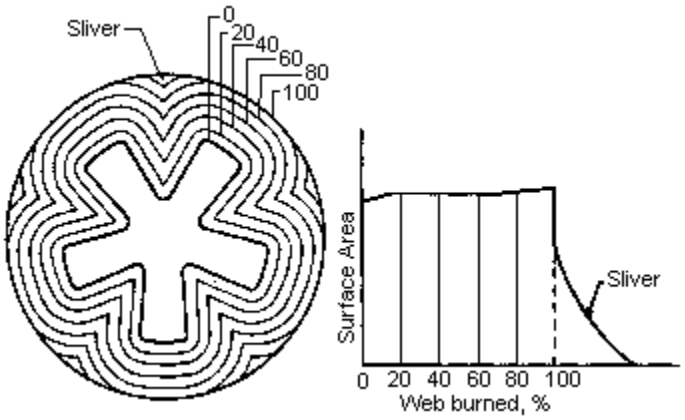

Figure 3 -- Grain regression

It is important to recognize that the **burning area of a propellant grain is a key parameter in determining the performance of a rocket motor**. The primary function of a propellant grain is to produce combustion products at a prescribed flowrate defined by:

$$\dot{m}_g = A_b \rho_p r$$

where ρ_p is the propellant mass density, A_b is the burning area, and r is the propellant burn rate. A complete discussion on burn rate is provided in the Propellant Burn Rate web page. The total burning area consists of all propellant surfaces that are exposed to combustion (and thus not inhibited from burning by some means). The grain burning area is dependant upon:

- Grain geometry, as described above
- Use of inhibitors

An *inhibitor* is a material or coating that is sufficiently heat resistant such that any propellant surfaces protected by the inhibitor do not combust during the entire operating duration of the motor. Inhibitors for amateur experimental motors are typically paper or cardboard, or a coating such as polyester or epoxy resin.
For the *design* of a motor, we are most interested in the *maximum* burning area, since it is this area that determines the *maximum chamber pressure* that the motor will experience. The maximum chamber pressure is used to size the motor casing. For a completely unrestricted-burning grain (e.g. A-100, B-200 & C-400 motors), all surfaces are exposed to the heated gases and thus burning proceeds from all surfaces commencing at the beginning of the burn. A "BATES" grain (Figure 4), which is

multiple-segment, hollow cylindrical grain that is case bonded or otherwise has the external surface inhibited, the initial burning surface is that area of the *core and segment ends*. The Kappa rocket motor utilizes such a grain configuration, with a total of four segments.

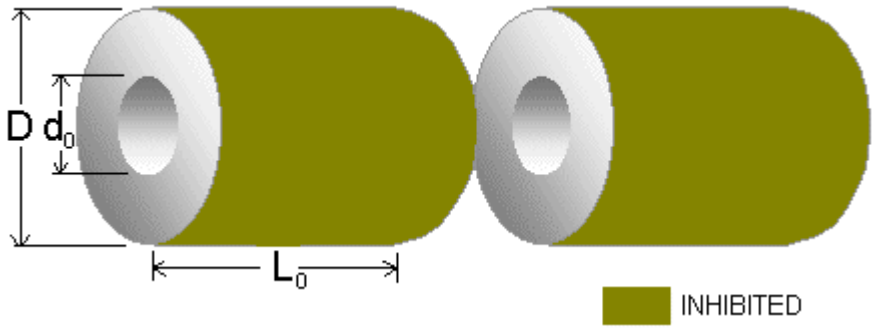

Figure 4 -- BATES grain

BATES grain configurations are often employed in amateur motors, as such a grain can be made to have approximately neutral burning, by choice of the appropriate Lo/D and D/do ratios. A more complete discussion on burning area and its relationship to grain geometry, and its influence on chamber pressure, is given in the Rocket Motor Design Charts -- Chamber Pressure web page.

The surface areas of a cylindrical grain *with frustum* (such as A-100, B-200 & C-400) are given in the Grain Area web page.

An important *physical property* of the propellant grain is the **Mass Density**, which is used in performance calculations. If a propellant is comprised of two constituents, and oxidizer and a fuel, the *ideal density* is given by

$$\rho_p = \frac{1}{\frac{f_o}{\rho_o} + \frac{f_f}{\rho_f}}$$

where the symbol ρ (rho) designates density, **f** is the mass fraction, and subscripts **o** and **f** refer to oxidizer and fuel , respectively. If a propellant is comprised of more than two constituents, then the ideal density is given by (where a, b, c... denote the individual constituents):

$$\rho_p = \frac{1}{\frac{f_a}{\rho_a} + \frac{f_b}{\rho_b} + \frac{f_c}{\rho_c} + \ldots}$$

In Table 1, the density for some oxidizers and some fuels is given, as well as a worked example. The *actual density* can be obtained by accurately weighing a grain to determine its mass, and by measuring its volume, with the density expressed as

$$\rho_p = \frac{m_{grain}}{\forall_{grain}} \qquad \forall_{grain} = \frac{\pi}{4}\left(D^2 - d^2\right)L$$

For a hollow cylindrical grain, where D =outer diameter
d =inner (core) diameter
L = Length of grain

The actual density will usually be some percentage less than the ideal density (typically 94%-97%), owing to tiny voids in the grain, and is dependant upon manufacturing technique. Volume is best obtained by the Archimedes principle, which involves immersion of the grain in an appropriate liquid, and measuring the displaced volume.

The **Volumetric Loading Fraction** is defined as the fraction of *grain volume* to *available chamber volume*, and relates the volumetric efficiency of the motor, as well as a measure of performance efficiency:

$$V_l = \frac{V_p}{V_a} = \frac{I_t}{I_{sp}\,\rho_p\,V_a}$$

where V_p is the grain volume, V_a is the available chamber volume, I_t is the total impulse (deliverable), and I_{sp} is the propellant specific impulse.

The **Web Fraction** is the ratio of propellant *web thickness* to grain *outer radius*, and is given by:

$$w_f = \frac{D-d}{D} = \frac{2\,r\,t_b}{D}$$

where t_b is the motor burn time. Clearly, to maximize burn duration, it is necessary to maximize the web fraction (i.e. thickness). The "price" for maximizing web thickness is reduction of the grain core diameter. This must be carefully considered, as explained below.

The **Port-to-Throat** area ratio is given by the *flow channel* cross-sectional area to the *nozzle throat* cross-sectional area:

$$\frac{A_p}{A_t} = \frac{\pi\,D^2\left(1 - V_l\right)}{4\,A_t}$$

where A_p is the flow (channel) area of the grain and A_t is the throat cross-sectional area. Gas velocity along the length of the flow channel is influenced significantly by the magnitude of the port-to-throat area ratio. *Choked flow* occurs when the ratio is 1.0, with flow velocity through the port being equal to the flow velocity through the nozzle throat (sonic). Severe erosive burning (core stripping) may occur under such a condition, and is generally avoided in design. The criticality of the port-to-throat ratio, however, depends upon the *mass flowrate* at a given location. In fact, a ratio of 1.0 (or less) may be used at the forward end of the grain where mass flowrate is minimum. The port-to-throat area ratio is often used as an index from which erosive burning tendencies are established. For those propellants where this has not been established, a ratio of 2.0 to 3.0 (dependant upon grain L/D ratio) is suggested.

Length-to-Diameter ratio is the g*rain overall length* in relation to the grain *outer diameter*. This parameter is very significant in motor design, as larger L/D values tend to result in greater erosive burning effects (including negative erosive burning). High L/D values tend to generate high mass flow rate differentials along the grain length, and may be best served with a *tapered* core or *stepped* core diameters (largest nearer the nozzle).

Next -- Propellant Combustion

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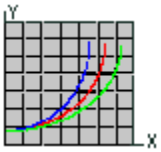

Solid Rocket Motor Theory -- Combustion

Propellant Combustion

A rocket motor operates on the basic principle of converting *heat energy*, from chemical reactions, to *kinetic energy*. In other words, the heat liberated by the combustion of propellant supplies the heat energy; the high velocity exhaust products exiting the motor have gained kinetic energy. This is why the exhaust experiences a significant drop in temperature as it flows through the nozzle (as is shown later), a requirement of the thermodynamics law of "conservation of energy".

Combustion is "simply" an exothermic chemical reaction. To get the 'ball rolling', an external heat source is required (igniter) to supply the necessary energy to a *threshold* level. This combustion is represented by a chemical equation. For example, for 65/35 O/F KN/Sucrose propellant, the combustion equation is given by (reactants -> products):

$$\mathbf{C_{12}H_{22}O_{11} + 6.29\ KNO_3 \rightarrow 3.80\ CO_2 + 5.21\ CO + 7.79\ H_2O + 3.07\ H_2 + 3.14\ N_2 + 3.00\ K_2CO_3 + 0.27\ KOH}$$

where the following compounds are symbolized as:

sucrose	solid	$C_{12}H_{22}O_{11}$
potassium nitrate	solid	KNO_3
carbon dioxide	gas	CO_2

carbon monoxide	gas	CO
steam	gas	H_2O
hydrogen	gas	H_2
nitrogen	gas	N_2
potassium carbonate	liquid	K_2CO_3
potassium hydroxide	liquid	KOH

Trace quantities of other compounds form, such as KH, and CH_4, but these can be ignored with little consequence.

Derivation of the complete combustion equation is potentially the most complex step in the analysis of a rocket motor. The propellant is burned, at (assumed) constant pressure, and forms a set of molecular products that are in thermal and chemical equilibrium with each other. The first step is to assume what the products of combustion *might* be. For propellants containing only carbon, oxygen, hydrogen, and nitrogen (C,H,O & N) there are (at least) twelve possible products -- carbon, carbon dioxide, carbon monoxide, hydrogen, steam, oxygen, nitrogen, nitric oxide as well as the dissociation products H, O, N and OH. If the propellant contains metallic elements such as potassium (K), sodium (Na), or aluminum (Al), or contains Chlorine (Cl) , this will result in condensed (liquid or solid) products of combustion, such as potassium carbonate, (or sodium equivalents), aluminum oxide or potassium chloride (KCl).

Once a set of possible products has been arrived at, the next step is to determine the mole numbers (or fractions) that will result. The mole numbers are the coefficients in the chemical equation. For the example above, the mole numbers are 3.67, 5.19, 7.91, 3.09, 3.14 and 3.14 for CO_2, CO, H_2O, H_2, N_2 and K_2CO_3, respectively, for the combustion of the reactants, being 1 mole sucrose and 6.29 moles of KNO_3.

Determining the mole numbers is accomplished by simultaneously solving a set of equations relating the reactants and products with respect to the conditions of :

- Mass balance
- Chemical equilibrium conditions
- Energy balance

Mass balance is straightforward, and refers to the principal of *conservation of mass*. The number of moles of any given element (e.g. C,H,O,N) *before* a chemical reaction must be equal to that *after* a chemical reaction. In the example above, the number of moles of atoms in the reactants are:

12 C atoms, **22** H atoms, 3*6.29=**29.87** O atoms, **6.29** K atoms, and **6.29** N atoms,

and in the products:

3.67+5.19+3.14=**12** C atoms, 2*7.91+2*3.09=**22** H atoms, 3*6.29=**29.87** O atoms, and 2*3.14=**6.29** (rounded) K & N atoms. The requirement of mass balance is met.

Many reactants, when mixed in definite quantities, react to form products only, in a so-called *irreversible reaction*. An example is the burning of a propellant (hence the "->" symbol in the equation). In a *reversible reaction*, however, the process goes both ways. Reactants form into products at the same rate that products form into the original reactants. This is the type of reaction with which chemical equilibrium conditions of hot combustion products are concerned. For example, the reaction

$2\ H_2 + O_2 <-> 2\ H_2O$ is a reversible reaction *see Note 1*

But what determines the relative concentration of these constituents (i.e. whether the reaction will proceed more to the left or to the right in this equation)? For each equation like this, there is an *equilibrium constant (K_p)* associated with it that determines this. This constant is a function of the *temperature* at which the reaction is occurring, and is essentially independent of other physical conditions, such as pressure. Values for various K_p can be found in thermochemical tables, such as the JANAF tables.
For the general reversible reaction:

$$\nu_A A + \nu_B B \Leftrightarrow \nu_C C + \nu_D D$$

the chemical equilibrium equation is of the form:

$$K_p = \frac{y_C^{\nu_C}\ y_D^{\nu_D}}{y_A^{\nu_A}\ y_B^{\nu_B}} \left(\frac{P}{P_o}\right)^{\nu_C+\nu_D-\nu_A-\nu_B}$$

where **y** is the equilibrium mole fraction of the A,B,C and D constituents, ν is the coefficient for each constituent (2,1, and 2 in the above example for the constituents H_2, O_2, and H_2O).

The term P/Po represents the ratio of pressure at which the reaction occurs, and the reference (standard state) pressure. It should be noted that the equilibrium of the combustion gases is very sensitive to temperature. Products existing at a high combustion temperature are very different from those existing at a lower combustion temperature. At high temperatures (above 3000 K), *dissociation* of the products occurs, as thermal energy causes the products to break up into simpler and monatomic constituents, such as

$$H_2O <-> HO + 1/2\ H_2$$
$$O_2 <-> 2\ O$$
$$H_2 <-> 2\ H$$

At lower combustion temperatures, negligible quantities of these constituents form (e.g. for KN/Sucrose combustion). Dissociation consumes energy that would otherwise be available for conversion to kinetic energy of the exhaust, and tends to limit the combustion temperature.

With the stated assumption of adiabatic combustion (no heat is lost to the surroundings) and well as the assumption of no changes in potential or kinetic energy, the law of conservation of energy states that the *enthalpy of the reactants* is equal to the *enthalpy of the products*:

$$H_R = H_P$$

Enthalpy can be looked upon as the heat involved in a chemical reaction. Put in another way, consider a reaction involving **n** moles of each reactant (represented by subscript **i**), and **n** moles of each product (subscript **e**)

$$\sum_R n_i\left[h_f + \Delta h\right]_i = \sum_P n_e\left[h_f + \Delta h\right]_e$$

The above equation states that the sum of the enthalpy of formation (h_f) plus the change of enthalpy (Δh), times the number of moles (n) for each constituent, is equal for both the reactants and the products. Note that Δh represents the change in enthalpy from a reference temperature, typically 298K (25C). For this reason, Δh is equal to zero for the reactants if the initial temperature of the propellant is assumed to be at this temperature.
Both, h_f and Δh for reactants and products may be found in thermochemical tables, for example, the JANAF tables or the NIST Chemistry WebBook.

The above equation is particularly useful, as it allows us the means to calculate the combustion temperature, which is usually referred to as the *adiabatic flame temperature* (AFT). Also, from this equation, it can be seen that a larger heat of formation of the reactants (per unit mass) is desirable, as is a smaller heat of formation of the products, since this will result in greater Δh available for the products (thus higher AFT). Indeed, the presence of diatomic gases (e.g. H_2, N_2) in the products is generally desirable simply because the heat of formation of these gases is zero.

A worked example of calculating the combustion temperature for KN/Sucrose, 65/35 O/F ratio, is given in Appendix A

The above describes a complete set of information that is necessary in order to

determine the complete combustion process. We now have a series of unknowns parameters, and equations that will allow for solution of the unknowns. In summary, the unknown parameters are:

1. The mole numbers (coefficients) in the mass balance equation
2. The mole fractions in the chemical equilibrium equations
3. The Adiabatic Flame Temperature
4. Δh values, these being a function of temperature
5. The (chamber) pressure at which the reaction occurs

Needless to say, this is not a simple task. The only practical way to solve the combustion problem is a computer solution, that is, to utilize an iterative procedure. This involves first assuming a chamber pressure at which the combustion occurs (the results are only weakly dependant upon pressure). The iterations of solving the equations begin at an assumed combustion temperature (AFT). These two assumptions allow the mole numbers and mole fractions to be determined for those initial conditions. Based on these determined values, a new AFT is computed using the energy equation. This value of AFT is then used in the next iteration to compute mole numbers and mole fraction, and so on. Eventually, convergence is (hopefully) reached and the final solution obtained. In fact, several years ago I wrote such a computer program tailored to analyze the combustion of the KN/Sucrose propellant, at varying O/F ratios. Indeed, it was not a small undertaking, but worked out well in the end.

Fortunately, it is no longer necessary for the interested amateur rocketry enthusiast to have to write such a program for the combustion analysis of a particular propellant, as there is software available that will do this job admirably and with great flexibility -- such as PROPEP, GUIPEP, or CET, which will be discussed later.

Next -- Nozzle Theory

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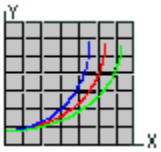
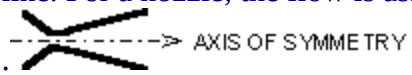


Solid Rocket Motor Theory -- Nozzle Theory

Nozzle Theory

The rocket nozzle can surely be described as the epitome of elegant simplicity. The primary function of a nozzle is to channel and accelerate the combustion products produced by the burning propellant in such as way as to maximize the velocity of the exhaust at the exit, to supersonic velocity. The familiar rocket nozzle, also known as a *convergent-divergent*, or *deLaval nozzle*, accomplishes this remarkable feat by simple *geometry*. In other words, it does this by varying the cross-sectional area (or diameter) in an exacting form.
The analysis of a rocket nozzle involves the concept of "*steady, one-dimensional compressible fluid flow of an ideal gas*". Briefly, this means that:

- The flow of the *fluid* (exhaust gases + condensed particles) is constant and does not change over time during the burn
- One-dimensional flow means that the direction of the flow is along a straight line. For a nozzle, the flow is assumed to be along the axis of symmetry



- The flow is *compressible*. The concept of compressible fluid flow is usually employed for gases moving at high (usually supersonic) velocity, unlike the concept of *incompressible* flow, which is used for liquids and gases moving at a speeds well below sonic velocity. A compressible fluid exhibits significant changes in density, an incompressible fluid does not.
- The concept of an ideal gas is a simplifying assumption, one that allows use of a direct relationship between pressure, density and temperature, which are properties that are particularly important in analyzing flow through a nozzle.

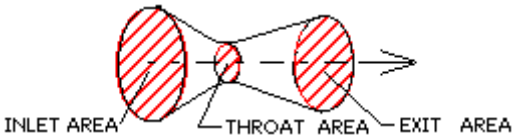
Fluid properties, such as velocity, density, pressure and temperature, in compressible

fluid flow, are affected by

1. cross-sectional area change
2. friction
3. heat loss to the surroundings

The goal of rocket nozzle design is to accelerate the combustion products to as **high an exit velocity as possible**. This is achieved by designing the necessary nozzle geometric profile with the condition that *isentropic flow* is to be aimed for. Isentropic flow is considered to be flow that is dependant only upon *cross-sectional area* -- which necessitates frictionless and adiabatic (no heat loss) flow. Therefore, in the actual nozzle, it is necessary to minimize frictional effects, flow disturbances and conditions that can lead to shock losses. In addition, heat transfer losses are to be minimized. In this way, the properties of the flow are near isentropic, and are simply affected **only by the changing cross-sectional area as the fluid moves through the nozzle**.

Typical nozzle cross-sectional areas of particular interest are shown in the figure below

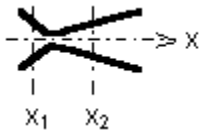


The analysis of compressible fluid flow involves four equations of particular interest:

1. Energy
2. Continuity
3. Momentum
4. The equation of state

The energy equation is a statement of the principle of conservation of energy. For adiabatic flow between any two points, x_1 and x_2 , it is given by

$$h_1 - h_2 = \tfrac{1}{2}\left(v_2^2 - v_1^2\right) = C_p\left(T_1 - T_2\right)$$



where *h* represents enthalpy of the fluid (which can be considered the energy available for heat transfer), *v* is the flow velocity in the x-direction, C_p is the effective *heat capacity* of the fluid, and *T* is the fluid temperature.
This equation provides valuable insight into how a rocket nozzle works. Looking at the first two terms shows that the change (decrease) in enthalpy is equal to the change (increase) in kinetic energy. In other words, heat of the fluid is being used to

accelerate the flow to a greater velocity. The third term represents the resulting change (decrease) in temperature of the flow. The heat capacity may be approximated to be constant, and is a property determined by the composition of the combustion products.

It is apparent, then, that the properties of a fluid (e.g. temperature) are a function of the flow velocity. In describing the state of a fluid at any point along its flow, it is convenient to consider the *stagnation state* as a reference state. The stagnation properties may be considered as the properties that would result if the fluid were (isentropically) decelerated to zero velocity (i.e. stagnant flow).

The *stagnation temperature*, To, is found from the energy equation (by setting v_2=0) to be

$$T_o = T + \frac{V^2}{2 C_p} \qquad \text{equation 1}$$

For an isentropic flow process, the following important relationship between stagnation properties for Temperature, Pressure, and Fluid Density hold

$$\frac{T_o}{T} = \left(\frac{P_o}{P}\right)^{\frac{k-1}{k}} = \left(\frac{\rho_o}{\rho}\right)^{k-1} \qquad \text{equation 2}$$

where ***k*** is the all-important **ratio of specific heats**, also referred to as the **isentropic exponent**, defined as

$$k \equiv \frac{C_p}{C_v} = \frac{C_p}{C_p - R}$$

Both C_p and *R* (specific gas constant) are properties determined by the composition of the combustion products, where R = R'/ M, where *R'* is the *universal gas constant*, and *M* is the *effective molecular weight* of the combustion products. If the combustion products contain an appreciable percentage of condensed phase particles (smoke), the value of the effective molecular weight, *M*, must account for this. As well, the proper *k* must be used which takes into account two-phase flow. The determination of *k* and *M* for the combustion products is detailed in the Technical Notepad #1 Web Page.

The local sonic velocity, *a*, and the Mach number, *M*, (defined as the ratio of the flow velocity to the local sonic velocity), is given by

$$a = \sqrt{k R T} \qquad M = \frac{V}{a} \qquad \text{equation 3}$$

From equations 1,2 & 3, the relationship between the stagnation temperature (also

referred to as *total temperature*) and Mach number may be written as

$$\frac{T_o}{T} = 1 + \frac{k-1}{2} M^2 \qquad \text{equation 4}$$

It can be shown from the first and second laws of thermodynamics, for any isentropic process, that

$$\frac{P}{\rho^k} = \text{constant} \qquad \text{equation 5}$$

From equations 4 & 5, and from the equation of state for an ideal gas, $P = \rho R T$, the relationship between stagnation pressure; density and Mach number may be expressed as given in the following two equations

$$\frac{P_o}{P} = \left(1 + \frac{k-1}{2} M^2\right)^{\frac{k}{k-1}} \qquad \text{equation 6}$$

$$\frac{\rho_o}{\rho} = \left(1 + \frac{k-1}{2} M^2\right)^{\frac{1}{k-1}} \qquad \text{equation 7}$$

Equations 4, 6 & 7 are particularly useful, as these allow each property to be determined in a flow if the Mach number and the stagnation properties are known. The stagnation (or total) properties To, Po, and ρ_o are simply the properties that are present in the combustion chamber of the rocket, since the flow velocity is (considered to be) zero at this location. In other words, To is the combustion temperature of the propellant (AFT), Po is the chamber pressure, and ρ_o is the density of the combustion products under chamber conditions.

Another important stagnation property is the *stagnation enthalpy*. This is obtained from the energy equation (by setting v_2=0)

$$h_o = h + \frac{v^2}{2} \qquad \text{equation 8}$$

Physically, the stagnation enthalpy is the enthalpy that would be reached if the flow (at some point) were somehow decelerated to zero velocity. It is useful to note that the stagnation enthalpy is **constant** throughout the flow in the nozzle. This is also true of the other stagnation properties (temperature, pressure, and density).

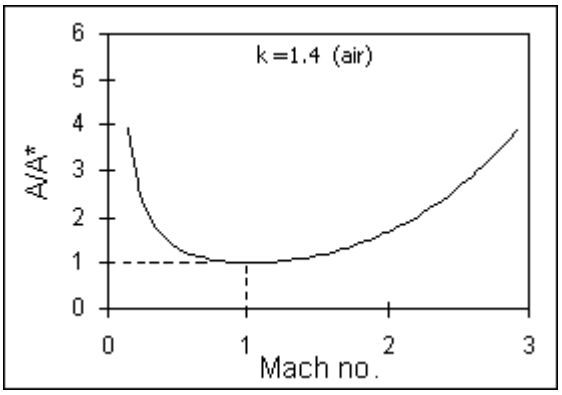
The second of the four equations of interest regarding compressible fluid flow, as discussed earlier, is the continuity (or conservation of mass) equation, which is given by

$$\rho A v = \text{constant} = \rho^* A^* v^* \qquad \text{equation 9}$$

where *A* is the nozzle cross-sectional area, *v* is the velocity of the flow. This equation simply states that the mass flowing through the nozzle must be constant. The "star" (asterisk) signifies a so-called *critical* condition, where Mach number is unity, M=1 (flow velocity is equal to the speed of sound). The importance of the critical condition will soon be made apparent.

Taking equations 3, 4, 7 & 9, it is possible to express the area ratio, A/A*, in terms of the Mach number of the flow. The area ratio is simply the cross-sectional area at any point (x) in the nozzle, to the cross-sectional area where the critical condition exists (M=1)

$$\frac{A}{A^*} = \frac{1}{M}\left(\frac{1 + \frac{k-1}{2} M^2}{1 + \frac{k-1}{2}}\right)^{\frac{k+1}{2(k-1)}} \qquad \text{equation 10}$$



When a plot is made of A/A* versus Mach number, using this equation, a very interesting result is obtained! It clearly shows that a *converging-diverging* passage with a section of *minimum area* is required to accelerate the flow from subsonic to supersonic speed. The critical point where the flow is at sonic velocity (M=1 at A/A*=1) is seen to exist at the *throat* of the nozzle. This shows the importance of the nozzle having a diverging section -- without it, the flow could never be greater than sonic velocity!

Supersonic flow is attained only through the diverging portion of the nozzle. Since the Mach number can be determined by knowing the area ratio, it is now possible to plot the variation of the temperature, pressure and fluid density throughout the nozzle, by use of equations 4, 6 & 7. A plot of these properties is given in Appendix C, for the Kappa nozzle.

From equations 8 & 9, the flow velocity at the nozzle exit can be expressed as

$$V_e = \sqrt{2\left(h_x - h_e\right) + V_x^{\,2}} \qquad \text{equation 11}$$

where subscripts *e* and *x* signify *exit* and any point *x* along the nozzle axis, respectively. This equation can then be put into the far more useful form with the aid of the energy equation and the definition of *k*, as well as equation 2.

$$V_e = \sqrt{2\,T_o\left(\frac{R'}{M}\right)\left(\frac{k}{k-1}\right)\left[1-\left(\frac{P_e}{P_o}\right)^{\frac{k-1}{k}}\right]} \qquad \text{equation 12}$$

This equation is one of the most useful, as it allows the **nozzle exit velocity** to be calculated. In summarizing, it is necessary to know

- *k*, effective ratio of specific heats of the exhaust products, obtained from the combustion analysis. For the condition of two-phase flow, the value must be modified, as explained in the *Two-Phase Flow* Theory Web Page.
- *R'* is the universal gas constant (R' = 8314 N-m/kmol-K)
- *M* is the effective molecular weight of the exhaust products, obtained from the combustion analysis, and must take into account the presence of all condensed-phase species.
- T_o is the combustion temperature of the propellant, also obtained from the combustion analysis
- P_e and P_o are the nozzle exit pressure and the chamber pressure, respectively. For most amateur rockets, P_e can be taken as ambient atmospheric pressure: Pe = Pa =1 atmosphere. P_o may be the measured chamber pressure, design chamber pressure, or the calculated chamber pressure (see "Chamber Pressure" section of Theory Pages).

A better understanding of the nozzle behaviour may be obtained by looking closely that this equation. It may be seen that

- Maximum exhaust velocity is obtained when exhausting into a vacuum (P_e = 0). This is the so-called *infinite pressure ratio, Po/Pe*.
- Increasing the chamber pressure does *not* significantly increase the exhaust velocity. If k=1.2, then it is found that doubling P_o from 35 atm. (515 psia) to 70 atm (1030 psia) increases the exhaust velocity by only about 7%.
- A *higher combustion temperature* and a *lower effective molecular weight* are both significantly and equally beneficial, being proportional and inversely proportional to the square root power, respectively.
- Although not obvious by looking at this equation, the effect of changing the

value of *k* is not too significant. A change from k=1.1 to k=1.2 results in a velocity loss of about 7%.

The ratio between the throat area, A*, and any downstream area in the nozzle, A_x, at which pressure P_x prevails can be conveniently expressed as a function of the pressure ratio, P_x/P_o, and *k*. By noting that at the throat M is unity, and using equations 2, 3, 4, 7 & 12, leads to

$$\frac{A^*}{A_x} = \left(\frac{k+1}{2}\right)^{\frac{1}{k-1}} \left(\frac{P_x}{P_o}\right)^{\frac{1}{k}} \sqrt{\left(\frac{k+1}{k-1}\right)\left[1-\left(\frac{P_x}{P_o}\right)^{\frac{k-1}{k}}\right]} \qquad \text{equation 13}$$

This is another important and useful equation. It allows the exit area, A_e, to be calculated such that the exit pressure, P_e, is equal to the ambient pressure, Pa (typically 1 atm.), by simply substituting P_a for P_x.

$$\frac{A^*}{A_e} = \left(\frac{k+1}{2}\right)^{\frac{1}{k-1}} \left(\frac{P_e}{P_o}\right)^{\frac{1}{k}} \sqrt{\left(\frac{k+1}{k-1}\right)\left[1-\left(\frac{P_e}{P_o}\right)^{\frac{k-1}{k}}\right]} \qquad \text{equation 14}$$

This is known as the **nozzle design condition**. For such a condition *maximum thrust is achieved* (derivation). For this design, the area ratio A_e/A^* is known as the all-important **Optimum Expansion Ratio**.

For a highly informative explanation on convergent-divergent nozzle operation, in particular choked flow and shock formation, visit the *Nozzle Applet* website (includes a simulation).

Next -- Rocket Motor Thrust

Last updated December 15, 2006

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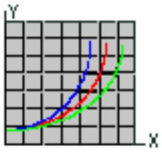


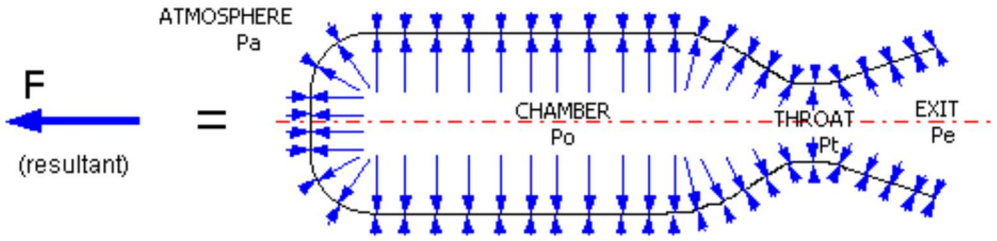
Solid Rocket Motor Theory -- Thrust

Rocket Motor Thrust and theThrust Coefficient

The *thrust* that a rocket motor generates is the most fundamental yardstick of performance. Without a doubt, this parameter is foremost in the mind of any amateur rocket motor designer. Thrust, being the *force* that a motor exerts, is what propels a rocket into (and beyond) the "wild blue yonder"!
Thrust is generated by the expelling of mass (the exhaust) flowing through the nozzle at high velocity. The expression for thrust is given by

$$F = \int P\,dA = \dot{m} v_e + (P_e - P_a) A_e \qquad \text{equation 1}$$

where the left hand term in the equation represents the *integral of the pressure forces* (resultant) acting on the chamber and nozzle, projected on a plane normal to the nozzle axis of symmetry, as shown in the figure.



The internal pressure is highest inside the chamber and decreases steadily in the nozzle toward the exit. External (atmospheric) pressure is uniform over the outside surfaces.

In the first term on the right-hand side of the equation, m is the mass flowrate of the exhaust products and v_e is the exhaust velocity. The second term on the right-hand side is the so-called *pressure thrust*, which is equal to zero for a nozzle with an optimum expansion ratio (Pe=Pa); A_e is the nozzle exit area.

Considering continuity (conservation of mass) at the nozzle throat, equation 1 may be rewritten as

$$F = \rho^* A^* v^* v_e + (P_e - P_a) A_e \qquad \text{equation 2}$$

This expression can now be modified using some equations that were presented in the Nozzle Theory Web Page, that is, the expressions for

- Fluid density ratio (noting that at the throat M=1), ρ_o / ρ (eqn. 7)
- Critical (throat) flow velocity, v* (eqn. 3, noting that v*=a)
- Nozzle exit velocity, v_e (eqn.12)
- and the equation of state for an ideal gas, $P = \rho R T$

gives

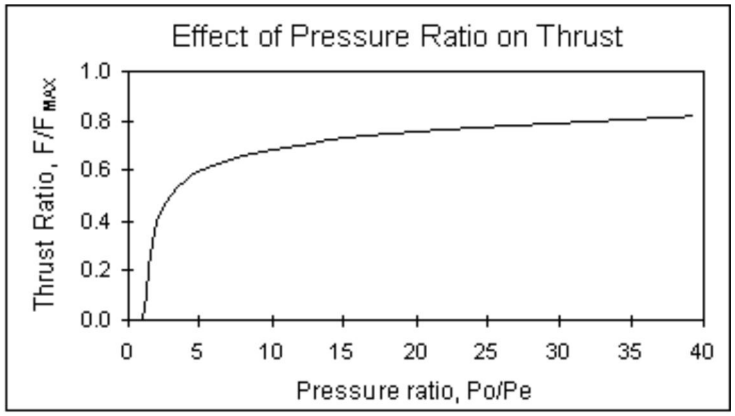
$$F = A^* P_o \sqrt{\frac{2k^2}{k-1}\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}\left[1-\left(\frac{P_e}{P_o}\right)^{\frac{k-1}{k}}\right]} + (P_e - P_a) A_e$$

equation 3

This equation shows that, if the pressure thrust term is zero, **thrust is directly proportional to throat area**, A*, and is **nearly directly proportional to chamber pressure**, Po.

This is particularly interesting. This means that if the throat size is doubled, the thrust will be doubled (if the chamber pressure is maintained). The same holds for the chamber pressure -- if it is doubled, thrust is approximately doubled. In reality, things are not so simple, as throat size and chamber pressure are tied together, as will be explained in the Theory Page on Chamber Pressure. This means that doubling a throat size would likely involve significant design changes, such as an increase in grain burning area. Likewise, if pressure is to be increased, the casing would have to be made stronger.
Thrust is also seen to be proportional to

- Pressure thrust (additive term, may be positive or negative)
- Ratio of specific heats, *k*. The sensitivity to *k* is quite low. For example, the difference in calculated thrust for k=1.4, compared to k=1.0, is a decrease of 14% (for a pressure ratio of Po/Pe=68).
- Pressure ratio across the nozzle, Pe/Po, as shown in the chart:



This chart plots the thrust ratio, F/F_{max}, to the pressure (or *expansion*) ratio, where F_{max} is the thrust that could be obtained with an infinite expansion ratio (i.e. expanding into a vacuum, with Pe=0). In the chart, the indicated thrust, *F*, excludes the pressure thrust term.The total thrust produced is given by $F_{total} = F + (Pe-Pa)\,Ae$.

The pressure ratio of the nozzle is determined solely by the area ratio, A*/Ae, as given by equation 14 of the **Nozzle Theory** page. What does this plot tell us?

- If the pressure ratio (and thus expansion ratio) is 1, then F = 0. The only thrust produced by such a nozzle is the pressure thrust, or $F_{total} = (Pe-Pa)Ae$. Such a nozzle, of course, would have no divergent portion, since A*/Ae=1, and would be a badly designed rocket nozzle!

- The *slope* of the curve is very steep initially, then begins to flatten out beyond Po/Pe = 5. This is significant, as it indicates that even a nozzle provided with a minimal expansion will be of significant benefit. With such a pressure ratio of 5, the resulting thrust is about 60% of maximum theoretical. From equation 14, it is found that the required area expansion ratio is only Ae/A* = 1.38 (for k=1.2), which translates to a required nozzle exit-to-throat diameter ratio of less than 2 !

The degree to which the thrust is amplified by the nozzle is quantified by the **Thrust Coefficient,** Cf, and is defined in terms of the chamber pressure and throat area:

$$F = C_f\ A^*\ P_o \qquad \text{equation 4}$$

The Thrust Coefficient determines the amplification of thrust due to gas expansion in the nozzle as compared to the thrust that would be exerted if the chamber pressure acted over the throat area only. Equation 4 is useful, as it allows for the experimental value of Cf to be obtained from measured values of chamber pressure, throat diameter, and thrust. The ideal value of Cf is calculated from equations 3 & 4, and shown below as equation 5:

$$C_f = \sqrt{\frac{2k^2}{k-1}\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}\left[1-\left(\frac{P_e}{P_o}\right)^{\frac{k-1}{k}}\right]} + \frac{(P_e - P_a)A_e}{P_o A^*}$$

equation 5

A KN/Sucrose motor equipped with a well designed nozzle will deliver a Cf of about 1.5 under steady-state conditions. Ideal Cf for the same motor would be around 1.65. A large fraction of the loss is due to two-phase flow inefficiencies.

As a final note, it should be pointed out that the equations for thrust and Cf (eqns. 3 & 5) require that *k* be corrected for two-phase flow.

Next -- Rocket Motor Impulse

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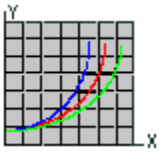

Solid Rocket Motor Theory -- Impulse and C-star

Total Impulse

Although *thrust* is an important yardstick for characterizing the *lift capability* of a rocket motor, it provides no indication of the how high the rocket will be propelled. For this, one needs a measure of the *total output* in terms of propulsion capability.The essential yardstick for for this is the **Total Impulse** of the rocket motor, which incorporates the essential element of time, or thrust duration.

Total Impulse is defined as the time integral of the thrust over the operating duration of the motor:

$$I_t = \int_0^{t_b} F\,dt \qquad \text{equation 1}$$

and is represented by the area under the thrust-time curve:

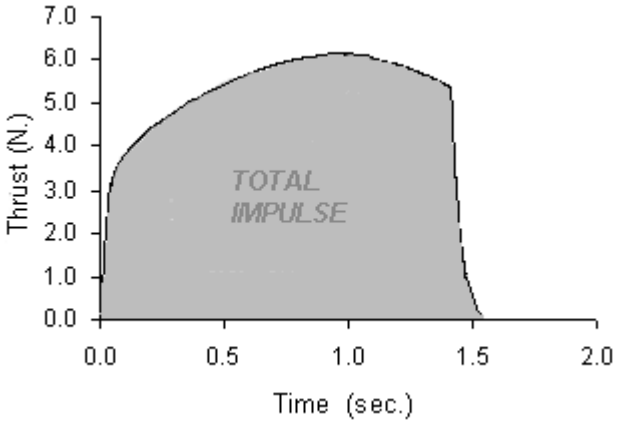

Figure 1 -- Thrust -time curve for a typical motor

Units are those of force multiplied by time, typically pound-seconds (lb-s) or Newton-seconds (N-s).
It is important to note that the Total Impulse only a tells part of the story regarding a motor's capacity to propel a rocket skyward. For example, a motor that delivers a Total Impulse of 200 lb-s may provide an average thrust of 100 lb. for 2 seconds (100 lb. x 2 s = 200 lb-s), or may deliver a thrust of 25 lb. for 8 seconds (25 lb x 8 s = 200 lb-s), as shown in Figure 2. Both deliver the same Total Impulse, which is usually abbreviated It.

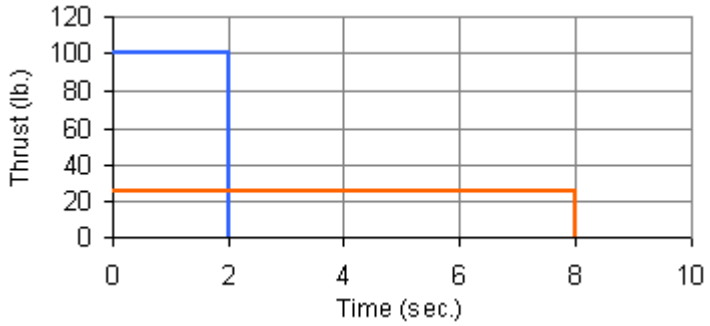

Figure 2 -- Two thrust-times curves with identical total impulse

The altitude achieved will differ to some extent, with this effect being more significant as the thrust/mass ratio drops. The more pronounced difference will be with the rocket's acceleration, since initial acceleration is given by:

$$a = F/m - g \qquad \text{equation 2}$$

where F = thrust, m = rocket liftoff mass, and g = acceleration of gravity. With lower acceleration, the longer it takes for the rocket to achieve a velocity at which the fins provide effective stability. And in the extreme case, if the thrust is less than the liftoff weight, the rocket will not even leave the launch pad, regardless of the motor's Total Impulse!

Characteristic Velocity

The *characteristic velocity*, also called *c-star* or simply *c**, is a figure of thermochemical merit for a particular propellant and may be considered to be indicative of the *combustion efficiency*. The expression for ideal c-star is given in equation 3, and is seen to be solely a function of the products of combustion (*k*, *M*, *To*).

$$c^* = \sqrt{\frac{R'/M \;\; To}{k\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}}} \qquad \text{equation 3}$$

The value used for *k* should be that for the mixture of gases and condensed phase, as shown in the Technical Notepad Web Page.

The delivered Specific Impulse is related to c-star as follows:

$$Isp = c^* C_f / g \qquad \text{equation 4}$$

where c* accounts for the influence of the combustion and C_f (thrust coefficient) accounts for the influence of the nozzle. As such, c* may be considered to be analagous to the specific impulse with a C_f =1.

The delivered c-star may be obtained from a rocket motor's pressure-time trace, being given by time integral of chamber pressure over the burn, multiplied by the ratio of throat area to propellant mass, as shown:

$$c^* = \frac{A_t}{m_p} \int_0^{t_b} P(t)\, dt \qquad \text{equation 5}$$

For the KN-Sugar motors, the deliverd c-star has been found to be in close agreement with the calculated value, indicating high combustion efficiency.

Specific Impulse

The **Specific Impulse** that a propellant is capable of producing (either theoretical or "delivered") is the key "yardstick" of performance potential. In its basic form, Specific Impulse can be considered to relate the *thrust produced* by a *unit mass* (e.g. 1 lb or kg) of propellant over a *burning time of one second*. As such, the units of Specific Impulse would be lb-s/lb or N-s/kg. In the former set of units, the "lb" can be considered to cancel, giving the more conventional units of "seconds". For the latter set of units, division of Specific Impulse in N-s/kg by the accleration of gravity, g (9.806 metre/s) results in the more conventional "seconds".

Delivered Specific Impulse produced by a motor, for example from static test measurements, is obtained from the expression:

$$Isp = I_t / w_p \qquad \text{equation 6}$$

where w_p is the propellant weight (lb or kg x g).
Delivered specific impulse has a dependancy upon:

- mass flowrate, and thus on motor size
- available combustion energy of the propellant
- nozzle efficiency
- ambient pressure conditions
- heat loss to the motor hardware

- two-phase flow losses
- combustion efficiency

These factors are discussed in detail the *Corrections for "Actual" Rocket Motors* Theory Web Page.

The **Ideal Specific Impulse** of a rocket propellant is calculated using equation 12 of the *Nozzle Theory* Web Page, which expresses exhaust velocity, Ve, in terms of the flow properties and the pressure ratio. Since Ve = c* Cf, ideal Isp can be determined from equation 4:

$$I_{sp} = \frac{1}{g}\sqrt{2\,T_o\left(\frac{R'}{M}\right)\left(\frac{k}{k-1}\right)\left[1-\left(\frac{P_e}{P_o}\right)^{\frac{k-1}{k}}\right]} \qquad \text{equation 7}$$

where k, M, To, Pe and Po are all defined in the Nozzle Theory Web Page. This equation is utilized to calculate the Ideal Specific Impulse for the KN/Sugar propellants, as shown in the *Technical Notepad* Web Pages.

Next -- Rocket Motor Chamber Pressure

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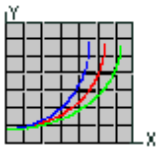


Solid Rocket Motor Theory -- Chamber Pressure

Chamber Pressure

The Chamber Pressure that a rocket motor develops is of crucial importance with regard to the successful operation of a rocket motor. Not only does Chamber Pressure strongly influence propellant burn rate, thermodynamic efficiency and thrust, the Chamber Pressure structurally loads the rocket motor casing and closures to a critical extent. Understanding the nature of Chamber Pressure generation, and accurate prediction of such, is one of the keys to successful rocket motor design.

What causes pressure to develop inside the chamber of a rocket motor? What determines the magnitude of this pressure? Intuitively, the pressure buildup is a result of the combustion of the propellant grain, whereby the gases produced hasten to escape through the nozzle throat. If the throat is sufficiently small, the gases cannot escape quickly enough and the accumulation of gases in the chamber results in pressurization.

In actuality, the intuitive explanation is essentially correct. However, an important factor that determines the *magnitude* of chamber pressure is not at all intuitive -- the concept of *choked flow*. This concept provides for a convenient means to calculate chamber pressure, and is valid for both transient and steady-state modes of motor operation, as discussed below.

By looking at a plot of Chamber Pressure over the operating duration of a rocket motor (Figure 1), one sees that there are three distinct and important phases of operation:

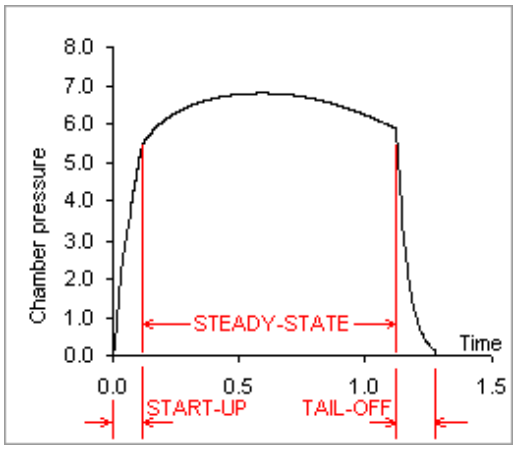


Figure 1 -- Motor chamber pressure

The pressure curve of the rocket motor exhibits *transient* and *steady-state* behaviour. The transient phases are when the pressure varies substantially with time -- during the ignition and *start-up* phase, and following complete (or nearly complete) grain consumption, when the pressure falls down to ambient level during the *tail-off* phase. The variation of chamber pressure during the steady-state burning phase is due mainly to variation of grain geometry (burning surface area) with associated burn rate variation. Other factors may play a role, however, such as nozzle throat erosion and erosive burn rate augmentation.

First of all, the start-up and steady-state pressure phases will be considered. The start-up phase is hypothetically very brief, although in reality, ignition of the complete grain does not occur instantaneously. The actual duration of the start-up phase is strongly dependant upon the effectiveness of the igniter system employed.
The steady-state phase clearly dominates the overall performance of the motor, and as such, constitutes the **design condition**.
In determining the start-up pressure growth, and the steady-state pressure level, it is first noted that the rate of *combustion product generation* is equal to the *rate of consumption of the propellant grain*, given by:

$$\dot{m}_g = A_b \rho_p r \qquad \text{equation 1}$$

where ρ_p is the propellant density, A_b is the grain burning area, and r is the propellant burn rate (surface regression rate).
It is important to note that the combustion products may consist of both gaseous and condensed-phase mass. The condensed-phase, which manifests itself as smoke, may be either solid or liquid particles. Only the gaseous products contribute to pressure development. The condensed-phase certainly does, however, contribute to the thrust (overall performance) of the rocket motor, due to its mass and velocity, as shown in equation 1 of the **Thrust Theory** Web page.

The rate at which combustion products are increasingly stored within the combustion chamber is given by:

$$\frac{dM_s}{dt} = \frac{d}{dt}(\rho_o \upsilon_o) = \rho_o \frac{d\upsilon_o}{dt} + \upsilon_o \frac{d\rho_o}{dt}$$ equation 2

where ρ_o is the instantaneous gas density in the chamber, and υ_o is the instantaneous gas volume (which is equal to the free volume within the chamber).

The change in gas volume with respect to time is equal to the change in volume due to propellant consumption, given by $d\upsilon_o/dt = A_b r$. This leads to:

$$\frac{dM_s}{dt} = \rho_o A_b r + \upsilon_o \frac{d\rho_o}{dt}$$ equation 3

The rate at which combustion products flow through the nozzle throat is limited by the condition of *choked flow*. As described in the **Nozzle Theory** Web Page, the flow achieves sonic (Mach 1) velocity at the narrowest portion of the convergent-divergent nozzle (throat). Flow velocity, at this location, can never exceed the local speed of sound, and is said to be in a *choked* condition. This allows us to determine the rate at which the combustion products flow through the nozzle is given by equation 4: (for derivation, see *Theory Appendix D*)

$$\dot{m}_n = P_o A^* \sqrt{\frac{k}{R T_o}} \left(\frac{2}{k+1}\right)^{\frac{k+1}{2(k-1)}}$$ equation 4

Note that R = R'/ M, where *R'* is the *universal gas constant*, and *M* is the *effective molecular weight* of the combustion products. Mass flow rate through the nozzle is seen to be a function of the chamber pressure (which determines the flow density), throat area, and the gas properties (which establish sonic velocity).

The principle of mass conservation requires the balance between *mass generation rate* and the sum of the rates at which *mass storage* in the chamber and *outflow through the nozzle*:

$$\dot{m}_g = \frac{dM_s}{dt} + \dot{m}_n$$ equation 5

Substituting equations 1 & 3 into equation 5 gives:

$$A_b \rho_p r = \rho_o A_b r + \upsilon_o \frac{d\rho_o}{dt} + \dot{m}_n$$ equation 6

Propellant burn rate may be expressed in terms of the chamber pressure by the Saint Robert's law (see *Propellant Burn Rate* Web Page):

$$r = a\,P_o^{\,n} \qquad \text{equation 7}$$

where *a* and *n* are the burn rate coefficient and pressure exponent, respectively. Substituting equations 7 & 4 (mass flowrate through nozzle) into equation 6 leads to the following equation:

$$A_b \rho_p\, a\, P_o^{\,n} = A_b \rho_o\, a\, P_o^{\,n} + \upsilon_o \frac{d\rho_o}{dt} + P_o A^* \sqrt{\frac{k}{R\,T_o}} \left(\frac{2}{k+1}\right)^{\frac{k+1}{2(k-1)}} \qquad \text{equation 8}$$

From the *ideal gas law*, the density derivative in the above equation may be expressed as:

$$\frac{d\rho_o}{dt} = \frac{1}{R\,T_o} \frac{dP_o}{dt} \qquad \text{equation 9}$$

As well, considering that chamber temperature, *To*, is essentially independent of chamber pressure, equation 8 may be re-written as:

$$\frac{\upsilon_o}{R\,T_o} \frac{dP_o}{dt} = A_b\, a\, P_o^{\,n} \left(\rho_p - \rho_o\right) - P_o A^* \sqrt{\frac{k}{R\,T_o}} \left(\frac{2}{k+1}\right)^{\frac{k+1}{2(k-1)}} \qquad \text{equation 10}$$

This is a particularly useful equation, as it allows us to determine the rate of change of chamber pressure (dP_o/dt) during the **transient *start-up* phase** of motor operation, where the chamber pressure is rapidly climbing up to the operating steady-state level. Once the steady-state phase is reached, when the outflow of combustion gases is in equilibrium with the production of gases from propellant consumption, $dP_o/dt = 0$, and the left-hand side of equation 10 vanishes. The steady-state chamber pressure may then be expressed as:

$$P_o = \left[\frac{A_b}{A^*} \frac{a\,\rho_p}{\sqrt{\frac{k}{R\,T_o}} \left(\frac{2}{k+1}\right)^{\frac{k+1}{(k-1)}}} \right]^{\frac{1}{(1-n)}} \qquad \text{equation 11}$$

Note that the combustion product density term has been dropped, as it is small in comparison to the propellant density.
Equation 11 may be greatly simplified by use of equation 7, letting $K_n = A_b / A^*$ and by noting that the characteristic exhaust velocity (c-star) is given by:

$$c^* = \sqrt{\frac{R\,T_o}{k\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}}}$$

This leads to the simplified expression for **steady-state chamber pressure**:

$$P_o = K_n\,\rho_p\,r\,c^* \qquad \text{equation 12}$$

where *r* is the burn rate at the chamber pressure, P_o.

The third and final phase of the pressure curve, the tail-down phase, ideally occurs immediately after the propellant grain has been completely consumed. In actuality, slivers or fragments of propellant grain remain once the bulk of the grain has been consumed. This results in a pressure tail-down that is more gradual than for the ideal case. However, it is impractical to account for this effect, and the tail-down pressure is determined on the assumption that the grain has been fully depleted.
After burnout , when $A_b = 0$, equation 10 becomes

$$\frac{\upsilon_o}{R\,T_o}\frac{dP_o}{dt} = -\frac{P_o\,A^*}{c^*} \qquad \text{equation 13}$$

This differential equation may then be solved to express **tail-off chamber pressure** as a function of bleeddown time for choked flow:

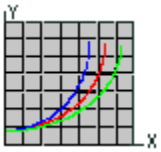
$$P_c = P_{bo}\,\exp\left(\frac{R\,T_o\,A^*}{\upsilon_o\,c^*}\,t\right) \qquad \text{equation 14}$$

where P_{bo} is the chamber pressure at burn-out and *t* is the time from burn-out. The pressure is seen to exhibit exponential decay.
In addition to the consequence of sliver burning during tail-off, *nozzle slagging* will tend to make the pressure decay more gradual than predicted by equation 14. Nozzle slagging is the tendency of condensed-phase (in particular liquid matter) to accumulate around the throat, effectively reducing the diameter. Slagging is more significant during tail-off due to the dropping pressure level and lower exhaust velocity.

An example of steady-state chamber pressure calculation, for the Kappa-DX motor, is provided in Theory Appendix E.

Next -- Two-phase Flow

Richard Nakka's *Experimental Rocketry* Web Site



Solid Rocket Motor Theory -- Two-phase Flow

Two-phase Flow

Most solid rocket propellants produce combustion products that are a mixture of gases and *condensed-phase* particles (either liquid or solid) which is evident as visible *smoke* in the exhaust plume. Those propellants containing metals, such as aluminum or magnesium, generate oxides of the metals as condensed-phase combustion products. Metallic-compound oxidizers, such as potassium nitrate (KN) or potassium perchlorate (KP), generate condensed-phase products of particularly high molecular weight, which is rather undesirable. The KN-Sugar propellants produce a dense white cloud of potassium carbonate smoke. In fact, approximately 44% of the exhaust mass is solid matter!

The occurrence of solids or liquids in the exhaust leads to a reduction in performance for a number of reasons:

- This portion of the combustion mass cannot perform any expansion work and therefore does not contribute to acceleration of the exhaust flow.
- The higher effective molecular weight of these products lowers the Characteristic Velocity (c*).
- Due to thermal inertia, the heat of the condensed-phase is partly ejected out of the nozzle before transferring this heat to the surrounding gases, and is therefore not converted to kinetic energy. This is known as **particle thermal lag**.
- Likewise due to the relatively large mass of the particles (compared to the gases), these cannot accelerate as rapidly as the surrounding gases, especially in that portion of the nozzle where flow acceleration is extremely high (throat region). Acceleration of the particles depends upon frictional drag in the gasflow, which necessitates a differential velocity.

The net result is that the condensed-phase particles exit the nozzle at a lower velocity than the gases. This is referred to as **particle velocity lag**.

In terms of the rocket performance parameters, the presence of condensed-phase products is reflected in a reduced Characteristic Velocity, due to the higher effective molecular weight of the gas/particle mixture.
The ideal Thrust Coefficient, C_f, on the other hand, is enhanced with increasing particle fraction, a consequence of a reduced *k* value. However, the delivered C_f suffers significantly, due to thermal lag and velocity lag. This is probably the largest single efficiency loss experienced by a motor with a significant fraction of particles in the exhaust. Such is especially true with a an underexpanded nozzle (e.g. divergent portion undersized). The apparent importance of having a good divergent portion of the nozzle is clear by examining Figure 3, which shows the variation of ideal Thrust Coefficient for flow through the Kappa rocket motor nozzle. The nozzle is designed with a near-ideal expansion ratio of $A_e/A_t = 11.4$, which gives an ideal Thrust Coefficient of 1.69. However, if the nozzle had been truncated at the throat (red dashed line), the coefficient would only be 0.62. The divergence therefore factors up the ideal thrust by a factor of 2.73 ! Of course, the "delivered" values are probably less pronounced than the ideal values. The delivered Thrust Coefficient for this motor is about 1.5. It is not known what the delivered Thrust Coefficient would be for a truncated nozzle, as this configuration has not been tested.

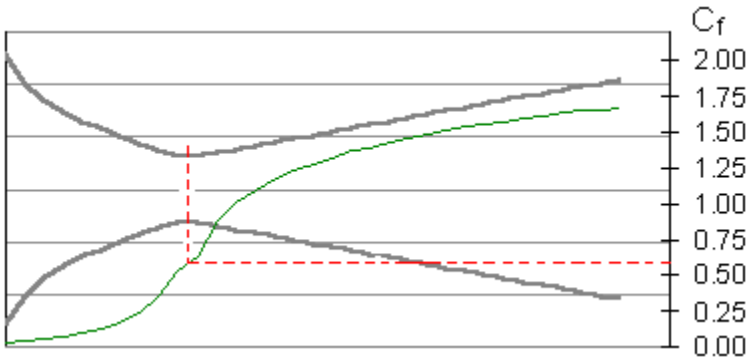


Figure 3 -- Ideal C_f for two-phase flow through Kappa nozzle

Another factor that is important with regard to two-phase flow losses is the *nozzle contour*, especially at the throat region. Figure 4 illustrates the flow acceleration for the Kappa nozzle. The acceleration in the region of the throat (red dashed line) is extremely high, especially just aft, where it is maximum. Most of the particle lag, which is a strong function of acceleration, occurs in this region, thus the importance of designing a nozzle with a well-rounded contour at the throat, without any sharp changes in cross-section.

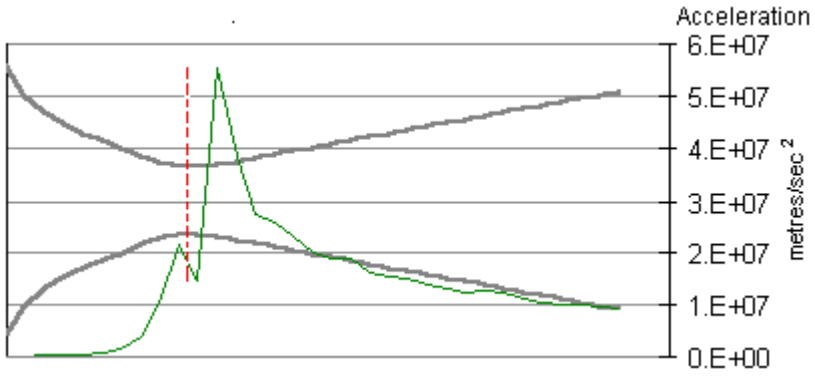


Figure 4 -- Gas/particle acceleration for two-phase flow through Kappa nozzle

The *size of the rocket motor* as well as *condensed-phase particle size* both play an important role with regard to the influence of two-phase flow effects. This is illustrated in Figure 5, which plots the fraction of Characteristic Velocity loss with respect to:

- Motor size (thrust)
- Particle size

Note that the mass fraction of particles in the exhaust for this study was X = 0.25. For the standard KN-Sugar propellants, X = 0.44.

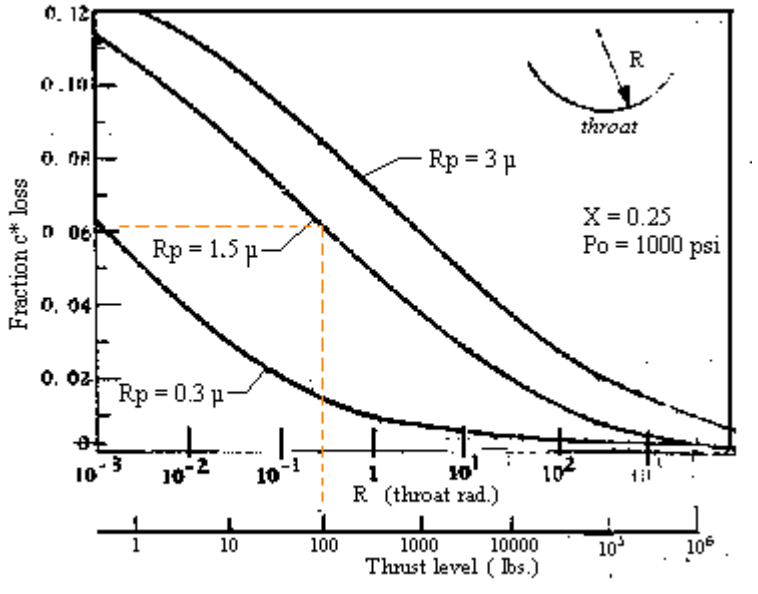


Figure 5 -- Influence of motor size and particle size on c*
Ref. Dynamics of Two-Phase Flow in Rocket Nozzles, ARS Journal, Dec.1962

For example, for a 100 lb. thrust motor, the motor suffers a 6% loss in Characteristic Velocity if the average particle size is 1.5 micron, as shown by the red dashed line.
It is clear from this plot that for amateur experimental motors, which are typically of 1000 lb. thrust or less, that two-phase flow losses can be significant, but can probably be disregarded for large "professional" motors.

How is two-phase flow taken into account with regard to motor performance calculations, such as those presented in the preceding Theory Pages? I asked myself that very question when I began researching the KN-Sucrose propellant from a theoretical performance aspect, back in 1983 when I began work on my B.Sc. thesis *Solid Propellant Rocket Motor Design and Testing*. All the equations in Sutton and other textbooks seemed to ignore the existence of particles in the exhaust, but I knew that I could not ignore this, not when the propellant exhaust contains 44% solid matter! After much consternation, I eventually managed to find a couple of books, and in particular, ARS Journal articles, that touched on this topic. I ended up re-deriving all the pertinent performance equations from basic principles, then made the necessary modifications to account for the presence of condensed-phase. A key assumption required was that the particles flow at the same velocity as the gas (i.e. no velocity lag), so the modified equations represent an upper limit on performance. The details are too involved to present here, so I will only present the final outcome, which fortunately, is quite simple. As it turns out, the gas-particle mixture behaves like a gas with a modified isentropic exponent, *k*. All the fundamental equations remain the same and are fully applicable to two-phase flow, with the only modifications being:

1. **Molecular Weight**, *M*, must take into account the presence of the condensed-phase by calculating the *effective Molecular Weight*, which is obtained by dividing the system mass by the number of moles of *gas* in the system.
   For example, if the system mass is 100 grams and the number of gas moles is 2.3819, then:

$$M = \frac{100}{2.3819} = 41.98 \quad \text{g/mole}$$

2. The modified **isentropic exponent** takes two forms, one for conditions where flow velocity (or actually, acceleration) is low, and the other for conditions of flow with high acceleration. Where flow acceleration is low, such as in the combustion chamber,

$$k = \frac{Cp_{mix}}{Cp_{mix} - R'} \qquad \text{equation 1}$$

where $C_{p_{mix}}$ is the *effective specific heat of the gas & solid mixture* and *R*' is the universal gas constant. The method of calculating $C_{p_{mix}}$ is provided in the Technical Notepad Web Page.
This is the form of *k* to be used, therefore, when calculating **chamber pressure** and **Characteristic Velocity**.

Where flow velocity and acceleration are high, that is, in the nozzle:

$$k = k' \left[\frac{1 + \psi \frac{Cs}{Cp_{gas}}}{1 + k' \psi \frac{Cs}{Cp_{gas}}} \right] \qquad \text{equation 2}$$

where *k'* is the isentropic exponent for the *gas only mixture* , $\psi = X/(1-X)$, where X is the mass fraction of particles in the exhaust. *Cs* is the specific heat for the solid (or liquid) mixture in the exhaust, and $C_{p_{gas}}$ is the specific heat for the gas only mixture.

The derivation of this form of modified isentropic exponent assumes a frozen flow condition where no thermal or velocity particle lag is assumed to exist, and is based on the momentum and energy equations for steady isentropic flow. Additional details on the calculation of this modified isentropic exponent may be found in the Technical Notepad Web Page and in the ARS Journal article "*Recent Advances in Gas-Particle Nozzle Flows*", R.F. Hoglund, May 1962.
This is the form of *k* to be used, therefore, when calculating **Exhaust velocity, Thrust, Thrust Coefficient** and the other nozzle flow parameters.

For those interested in more of the theoretical treatment of two-phase flow, I'd suggest perusing *Solid Propellant Rocket Motor Design and Testing* which is available for downloading in PDF format.

Next -- Corrections for Actual Motors

Last updated August 19, 2001

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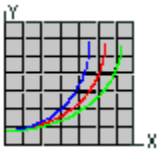


Solid Rocket Motor Theory -- Corrections for Actual Rocket Motors

The preceding Web Pages dealing with solid rocket motor theory consider the analysis of an *ideal rocket*, which of course, does not exist. The ideal rocket represents the maximum performance condition that could be attained if it were not for real-world factors and other approximations that lead to performance reductions in *actual* solid rocket motors. These are accounted for by using various **correction factors** in the design or analysis of a rocket motor.

Chamber Conditions

Combustion efficiency and heat losses through the chamber wall both tend to produce a lower chamber pressure than predicted by theory. Solid propellant, however, typically has a high combustion efficiency if well mixed and the oxidizer particle size is very fine. A measure of the combustion efficiency of a propellant can be taken by comparing the measured (delivered) value of characteristic velocity (cee-star) to the ideal value:

$$\eta^* = \frac{\bar{c}^*}{c^*}$$

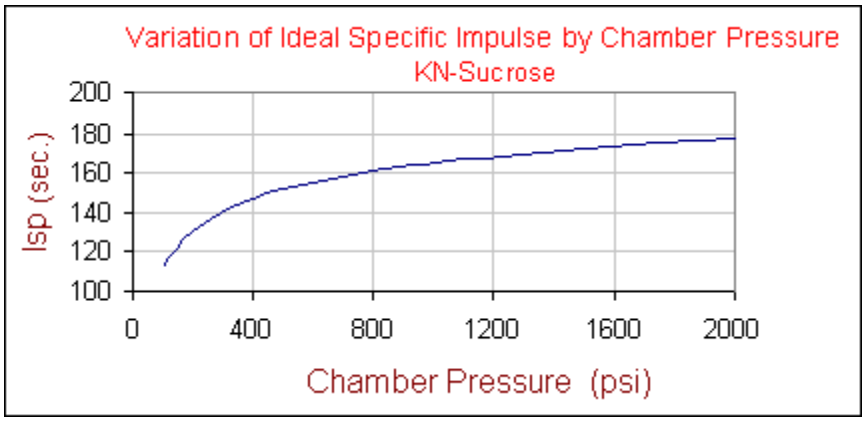
The delivered value of cee-star can be obtained from pressure measurements of static test results:

$$\bar{c}^* = \frac{A_t}{m_p} \int_0^{t_b} P(t)\, dt$$

or measured by "closed vessel" combustion of propellant samples.
For well-prepared sugar-based propellants, the combustion efficiency has been measured to be between 98 and 99%. To some degree, the combustion efficiency is a function of the motor size. Motors with longer combustion chambers provide more time for the chemical reactions to occur before dispelling through the nozzle.

Heat loss through (or into) the chamber walls is also dependant upon motor size, as well as casing material and wall thickness. For example, a larger sized motor with a thin-walled steel casing would have much less heat loss than a small motor with relatively thick walled aluminum casing. However, the overall detrimental effect is probably insignificant for both.

The chamber pressure has a pronounced effect upon the propellant specific impulse, particularly at the lower pressure regime, as shown in the figure below:



As amateur experimental rocket motors typically have short burn times, a significant portion of the total impulse may result from the pressure *start-up* or *tail-off* phases of the burn, when the chamber pressure is well below the steady-state operating pressure level. As a result, the total *delivered* specific impulse suffers. This is one reason why delivered specific impulse can be lower than ideal, which is based on constant steady-state pressure (usually referenced at 1000 psi). The extent of loss, designated ς_p, is highly dependant upon the motor burn time and pressure-time profile, but may be 5% or greater. Thus a typical pressure correction factor would be $\varsigma_p = 0.95$.

Nozzle Corrections

The flow through a real nozzle differs from that of an ideal nozzle because of frictional effects, heat transfer (particularly at the throat), imperfect gases and incomplete combustion, non-axial flow, nonuniformity of the fluid, and particle velocity and thermal lag.
Conical nozzles are used almost exclusively for amateur motors, due to the relative simplicity in manufacturing such a nozzle. In nozzle theory, flow is assumed to be one-dimensional (axial). In a conical nozzle, the flow is two-dimensional, with the extent of the non-axial velocity dependant upon the divergence cone half-angle, α. The correction factor for non-axial flow is given

by:

$$\lambda = \tfrac{1}{2}(1 + \cos\alpha)$$

This loss is usually quite small, with typical values being $\lambda = 0.99$ for a 12 degree half-angle and $\lambda = 0.97$ for a 20 degree half-angle.

The *discharge correction factor* is used to express how well the nozzle design permits the mass flow rate through the throat to approach the theoretical rate, and is given by the ratio of delivered mass flow rate to ideal mass flow rate:

$$\varsigma_d = \frac{\bar{\dot{m}}^*}{\dot{m}^*}$$

The most significant design parameter which determines the discharge factor is the contour at the entrance region of the throat. A well rounded contour tends to maximize the actual flow rate. For propellants that have a significant fraction of particles in the exhaust, good contouring minimizes acceleration of the flow at the entrance, thus minimizing the two-phase flow loss associated with particle velocity lag.
Certain factors tend to *increase* the actual mass flow rate in comparison to the idealized mass flow rate. These factors include

- heat transfer of the fluid to the nozzle walls, tending to decrease the flow temperature, increasing the density.
- the specific heat ratio and other gas properties change through the nozzle in such a way as to increase the discharge factor.

Consequently, for a rocket motor that has no condensed-phase products in the exhaust, the discharge correction factor may be close to unity. However, for a rocket motor that utilizes a propellant with a large fraction of condensed-phase products (such as the KN-Sugar), the losses can be quite significant, even with a well contoured nozzle entrance. The value of the discharge correction factor would typically be $\varsigma_d = 0.90$ for this propellant with a well designed nozzle with smooth flow surfaces and minimal heat loss.

Corrections for Specific Impulse

The Ideal Specific Impulse must be corrected to obtain the Delivered Specific Impulse of an actual rocket motor, by applying the correction factors discussed above:

$$\bar{I}_{sp} = \eta^* \varsigma_p \varsigma_d \lambda I_{sp}$$

As an example, the Kappa-DX rocket motor, powered by the KN/Dextrose propellant, has the following correction factors:

- Combustion efficiency correction factor $\eta^* = 0.98$
- Chamber pressure correction factor $\varsigma_p = 0.95$ (estimated)
- Nozzle discharge correction factor $\varsigma_d = 0.91$ (estimated)
- Nozzle divergence correction factor $\lambda = 0.99$

As the Ideal Specific Impulse is Isp = 164 sec. (@1000 psi), the Delivered Specific Impulse is given by:

$$I_{sp} = (0.98)\,(0.95)\,(0.91)\,(0.99)\,164 \quad = 138 \text{ sec.}$$

Next -- GUIPEP : Propellant Performance Software

Last updated August 19, 2001

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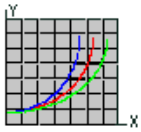


Solid Rocket Motor Theory -- GUIPEP

Introduction

This Web Page is intended to serve as an introduction to the GUIPEP software, which is basically PROPEP software (PC version of the Propellant Evaluation Program) with a Graphical User Interface (GUI) added to greatly simplify usage of the program. This highly useful thermochemical software allows the user to evaluate the theoretical performance of a solid (or liquid) rocket propellant. As such, it is particularly useful for checking the viability of possible propellant formulations. As well, it allows the user to quickly determine the most effective ratios of ingredients to achieve desired performance, from a theoretical perspective.

GUIPEP is primarily a *chemical equilibrium* solver, that is, it balances the chemical equations relating the propellant reactants and products by a method known as "minimization of Gibbs free energy". The ingredients (reactants) defining the propellant are transformed adiabatically and irreversibly to reactions product constituents in the amounts fixed by equilibrium relations, chamber pressure, and mass balance at a reaction temperature fixed by the available energy of reaction. The resulting set of products provides the basis for computation of thermodynamic properties from which performance parameters are determined by an iterative process to account for changing product properties and composition.

Input is simply a list of propellant ingredients (and the mass of each), as well as chamber pressure and nozzle exit pressure. Solver output includes combustion temperature, isentropic exponent, molecular weight of products, exhaust temperature and composition, specific impulse, and ideal expansion ratio. Note that burn rate parameters are *not* evaluated, as burn rate is a complex phenomenon that involves many other physical processes besides combustion, such as heat and mass transfer between the reaction flame and propellant burning surface.

Another similar thermochemical program is CET (Chemical Equilibrium with Transport Properties, NASA TM4557), but as far as I know, no GUI is available for this software. As such, it is cumbersome to use. The predictions are nearly identical to those of GUIPEP, based on my limited experience with using this program.

Analysis Assumptions

The basic assumptions employed by the solver are much as described in the Basic Assumptions Theory Web Page:

- One dimensional flow with regard to the continuity, energy and momentum equations
- Zero flow velocity at the nozzle inlet
- Complete and adiabatic combustion
- Isentropic expansion in the nozzle
- Homogeneous mixing of the reactants and products
- Ideal-gas law applies
- Zero temperature lag and velocity lag of the condensed-phase products

---

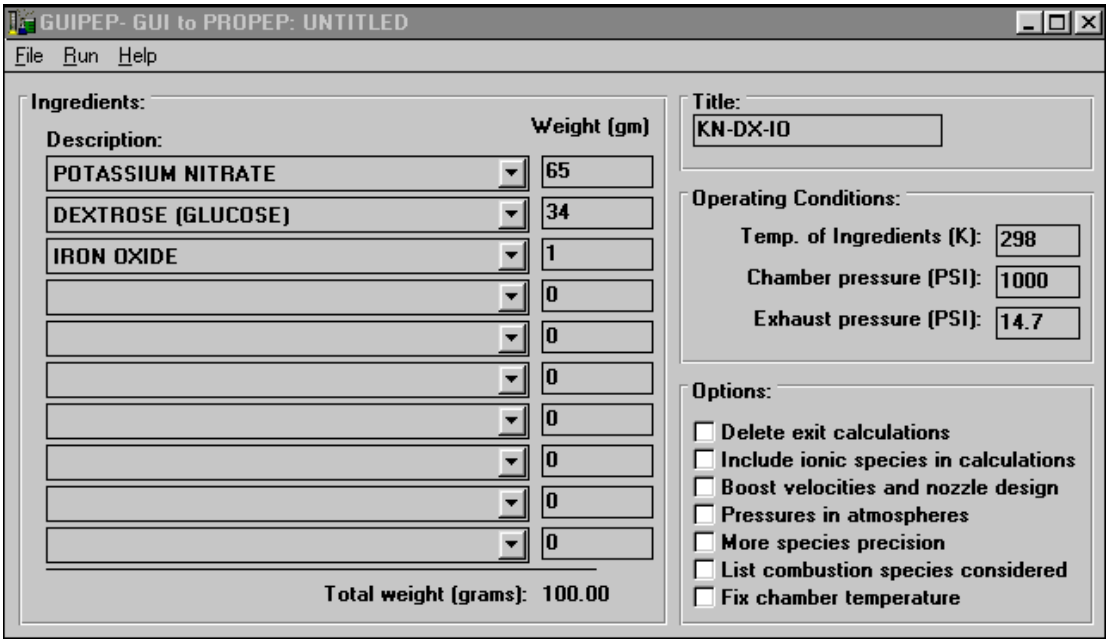
Using GUIPEP

GUIPEP is very easy to use. Up to 10 propellant ingredients are chosen from the drop-down boxes, and the mass (in grams) is entered. Total mass need not add up to 100 grams, but this is the most convenient way to enter the data, as the mass then represents the *percentage* of that particular constituent.
To eliminate any unwanted ingredient, zero is entered as the mass.
A **Title** of the run is then entered, and may be up to 10 characters in length.
The **Operating Conditions** are usually left as the default values, unless there is some particular reason to modify them:

- Temperature of ingredients = 298 K (which is room temperature, 25C.)
- Chamber pressure = 1000 psi (which is the reference pressure at which Isp is quoted).
- Exhaust pressure = 14.7 psi (which is one atmosphere, the condition of ideal expansion at sea level).

As far as **Options** are concerned, none need be chosen for basic propellant evaluation. However, if the nozzle design is being studied, check the *Boost Velocities and Nozzle Design* box.

The final step is to run the program by selecting *Run*, then *Single Run*. A DOS box then appears to allow execution of the program, which is initiated by hitting the *Enter* key. MS Notepad then appears, in which the output is displayed.
A screen-shot of an example GUIPEP input screen is shown below:



GUIPEP Output

The initial portion of the output is a basically an echo of the complete input data, as shown below:

```
File  Edit  Search  Help
■■ KN-DX-IO          Run using June 1988 Version of PEP,
 Case  1 of  1     11 Aug 2001 at  9:14:10.68 pm

  CODE                                  WEIGHT    D-H  DENS      COMPOSITION
   821 POTASSIUM NITRATE                65.000  -1169  0.07670    1N   3O   1K
  1093 DEXTROSE (GLUCOSE)               34.000  -1689  0.05670    6C  12H   6O
   541 IRON OXIDE                        1.000  -1230  0.18400    3O   2FE

THE PROPELLANT DENSITY IS 0.06884 LB/CU-IN OR  1.9056 GM/CC
THE TOTAL PROPELLANT WEIGHT IS  100.0000 GRAMS

NUMBER OF GRAM ATOMS OF EACH ELEMENT PRESENT IN INGREDIENTS

   2.264628 H     1.132314 C     0.642877 N     3.079730 O
   0.642877 K     0.012523 FE
```

Some of the input data is automatically pulled from the *pepcoded.daf* file, which is a text file that contains the following ingredient data:

- Ingredient name
- Chemical formula
- "Heat of formation" (which is actually delta enthalpy of formation), in calories/gram
- Mass density, in pounds/cubic inch

This data is reflected in the above output, where D-H is the "delta heat of formation", DENS is the constituent density, and COMPOSITION is the chemical formula. Resulting propellant ideal density is also given, and is computed according to the following equation:

$$\rho_p = \frac{1}{\frac{f_a}{\rho_a} + \frac{f_b}{\rho_b} + \frac{f_c}{\rho_c} + \ldots}$$ equation 1

as detailed in the Propellant Grain Theory Web Page.
e.g. DENS = 1/(0.65/0.0767 + 0.34/0.0567 + 0.01/0.184) = 0.06884 lb/in^3.

The number of gram-atoms of each element present in the ingredients is then listed. Basically, this indicates how many relative *atoms* of each element are present in the cauldron of ingredients that are combined to form the products of combustion. Although this is key information for the solver, for the user it serves no particular purpose. For reference, this is calculated as the mass to molecular weight ratio for a particular ingredient, multiplied by the mole number for a particular element, summed for each ingredient.

The next portion of the output presents the **combustion chamber conditions**, as shown below:

```
**************************CHAMBER RESULTS FOLLOW    ******************************

 T(K)  T(F)  P(ATM)   P(PSI) ENTHALPY  ENTROPY   CP/CV     GAS   RT/V
1733. 2659.   68.02  1000.00  -134.64   163.44  1.1280  2.297   29.614

SPECIFIC HEAT (MOLAR) OF GAS AND TOTAL=   10.801   15.381
NUMBER MOLS GAS AND CONDENSED=   2.2970   0.3179

  0.87508 H2O         0.41818 CO2         0.40865 CO          0.32138 N2
  0.30541 K2CO3*      0.24164 H2          0.03037 KHO         0.01242 FeO*
 1.30E-03 K          1.70E-04 K2H2O2     8.55E-05 FeH2O2     6.85E-05 NH3
 1.80E-05 H          1.05E-05 KH         4.87E-06 KCN        3.75E-06 HO
 2.13E-06 CH2O       2.12E-06 CH4        1.63E-06 CNH

THE MOLECULAR WEIGHT OF THE MIXTURE IS   38.243
```

The first row indicates the **combustion temperature** (in Kelvin and degrees F), the **chamber pressure** as specified, the total **enthalpy** of the mixture (kcal/system mass), total **entropy** of the system (cal/K/system mass), **CP/CV,** which is the ratio of specific heats, **GAS** (number of gas moles in the mixture), and **RT/V** (a conversion factor which is not normally used). Note that the system mass in this example is 100 grams.
The only important parameters here are:

- **Combustion temperature** - Also referred to as the *Adiabatic Flame Temperature*, and determined by the method described in the Combustion Theory Web Page. Generally, the higher the temperature, the higher the specific impulse. Two "real world" factors to consider, however. Higher temperatures require more robust casing and nozzle materials, insulation, or ablative coatings. Note that the chamber temperature is the *stagnation temperature* that the nozzle will "see" and must be designed for.
  Low combustion temperatures, as predicted by this program, may not be self-sustaining in reality. For example, a formulation with a predicted chamber temperature of 1000 K will probably not combust at all.

- **CP/CV** - The ratio of specific heats, *k*, for the mixture at combustion chamber conditions, this is correct value to use when calculating characteristic velocity (cee-star) and chamber pressure, as described in the preceding Theory Web Pages. The value of CP/CV is calculated from the following

equations:

$$k = \frac{Cp_{mix}}{Cp_{mix} - R'} \quad \textit{where} \quad Cp_{mix} = \frac{1}{n} \sum_i \left(n_i Cp_i + n_s C_s \right) \qquad \text{equations 2 \& 3}$$

with the details on notation and use of the equations provided in the Technical Notepad Web Page.

- **GAS** - The number of moles of *gaseous* combustion products in the product mixture (which may also contain condensed phase). This value is used to calculate the effective Molecular Weight, *M*, of the product mixture, which is given by dividing the number of gas moles into the system mass. For this example, M = 100/ 2.297 = 43.54 g/mole. This is the proper molecular weight value to use in the gas dynamics equations described in the preceding Theory Web Pages.

The next two lines provide the values of the **molar specific heat** of the gaseous products and of the mixture, (cal/mole/K), and are provided for reference only.
The following line provides the values for the number of **gas moles** (repeated) and the number of **moles of condensed phase** products, which may be solid or liquid. This information is of interest, as it provides the (molar) ratio of gas/condensed phase products.

The next lines of output tabulate the number of **moles of each combustion product constituent**. Product names followed by ***** are liquid phase, and **&** designates solid phase; all others are gas phase. This data allows the user to calculate the *mass fraction of condensed phase*, which is given by the mass of all condensed phase divided by the system mass, and where the mass of any constituent is given by the *number of moles* multiplied by the *molecular weight* of that constituent.
e.g. Mass fraction of condensed phase = [(0.30541) 138.2 + 0.01242 (71.9)]/ 100 = 0.422

Many of the combustion products are in trace amounts, and play a negligible role in the overall process. From the example above, the only significant products are H2O, K2CO3, CO2, H2, CO, N2 and perhaps KOH and FeO.
For best performance, low molecular weight products are desirable, such that the effective molecular weight of the mixture is minimized. Low molecular weight products in the above example would be H2O, H, H2, CH4, CO, NH3 and OH.

The next line in the above portion of the output gives the molecular weight of the mixture (sometimes denoted *MW*), which is given by the sum of the mole fraction, for each constituent, multiplied by its molecular weight, as shown below:

$$M_{mix} = \sum_i f_{mi} M_i \qquad \text{equation 4}$$

This value of molecular weight should be neglected as it serves no purpose with regard to rocket performance.

The next portion of the output presents the **nozzle exhaust conditions**, as shown below:

```
**************************EXHAUST RESULTS FOLLOW    ******************************

 T(K)  T(F)  P(ATM)   P(PSI) ENTHALPY  ENTROPY   CP/CV     GAS    RT/V
1169. 1646.    1.00    14.70  -161.60   163.44  1.1325  2.266    0.441

SPECIFIC HEAT (MOLAR) OF GAS AND TOTAL=    9.969    14.803
NUMBER MOLS GAS AND CONDENSED=   2.2656   0.3334

  0.77686 H2O         0.51521 CO2         0.35490 H2          0.32141 N2
  0.32090 K2CO3&      0.29614 CO          0.01250 FeO&        0.00101 KHO
 4.62E-05 K          1.17E-05 NH3        2.42E-06 CH4        1.75E-06 K2H2O2

THE MOLECULAR WEIGHT OF THE MIXTURE IS   38.475
```

The format of these results is identical to that of the chamber results. The values represent the conditions at the *exit plane* of the nozzle.

Some points worth noting:

- The combustion product temperature has dropped significantly, as thermal energy has been converted to kinetic energy. The exit temperature may be calculated from *equation 4* of the Nozzle Theory Web Page.
  :

$$T_e = \frac{T_o}{1 + \frac{k-1}{2} M_e^2} \quad \text{where} \quad M_e = \sqrt{\frac{2}{k-1}\left[\left(\frac{Po}{Pe}\right)^{\frac{k-1}{k}} - 1\right]} \quad \text{equations 5 \& 6}$$

  where *To* is the chamber temperature, *Po/Pe* is the chamber/exit pressure ratio, *Me* is the mach number of the flow at the exit, and *k* is the CP/CV for exhaust conditions. Note that the value given in the output is for conditions of *shifting equilibrium* which is explained later.

- Chamber pressure has dropped to one atmosphere, the design condition.
- Both CP/CV and the number of gas moles has changed slightly, reflecting the changing composition and temperature of the exhaust as it flows through the nozzle.
- Likewise, the specific heats and number of moles of condensed species has changed from chamber conditions.
- The composition of the products has changed in an interesting way. Note that there are fewer *trace* constituents. This is because the temperature is lower and less *dissociation* (breaking up into simpler molecules) of the larger compounds occurs. Also note that the liquid products have frozen into solid phase.

The next portion of the output presents the **Performance** of a rocket motor equipped with this propellant and nozzle as specified:

```
**********PERFORMANCE:  FROZEN ON FIRST LINE, SHIFTING ON SECOND LINE**********

IMPULSE   IS EX      T*      P*      C*     ISP*  OPT-EX  D-ISP      A*M   EX-T
  151.6  1.1326   1625.   39.31  2967.9           10.22  288.9  0.09227  1057.
  153.2  1.1058   1647.   39.63  3025.2   114.3   10.82  291.9  0.09405  1169.
```

Performance is given for both **Frozen** and **Shifting** equilibrium conditions.
What do these terms mean? Frozen equilibrium means that the chemical composition of the exhaust

does not change as it flows through the nozzle (product composition is established in the combustion chamber). Shifting equilibrium assumes that instantaneous chemical equilibrium is established as the gas expands through the nozzle, "shifting" the composition continuously.
Why are both results provided? Because of the very short residence time in the nozzle, it is uncertain whether or not there is sufficient time for chemical reactions to actually occur as predicted by the shifting equilibrium model. Geometry also plays a role, as longer nozzles provide more residence time.
Which results to use? For amateur motors, where nozzles are very small in comparison with large professional rockets, I consider the frozen flow model to be more realistic. For the Kappa rocket motor nozzle, I've calculated the time duration for the flow to pass through the nozzle to be 430 microseconds!

In the performance portion of the output, the first row presents the ideal Specific Impulse (IMPULSE), isentropic exponent (IS EX), flow temperature at the throat (T*) and pressure at the throat (P*), characteristic velocity (C*), vacuum impulse (ISP*), optimum expansion ratio (OPT-EX), density Isp (D-ISP), throat area-to-mass flow rate (A*M), and exit-plane temperature (EX-T).

The following is a brief discussion of each of the results:

- **Ideal Specific Impulse** is the key "yardstick" of performance potential, and can be considered to relate the *thrust produced by a unit mass* (e.g. 1 lb or kg) of propellant over a *burning time of one second*. The ideal Specific Impulse may be determined from equation 7 of the Impulse and C* Theory Web Page:

$$I_{sp} = \frac{1}{g}\sqrt{2\,T_o\left(\frac{R'}{M}\right)\left(\frac{k}{k-1}\right)\left[1-\left(\frac{P_e}{P_o}\right)^{\frac{k-1}{k}}\right]} \qquad \text{equation 7}$$

  where *k* is taken as the average of CP/CV for chamber and exhaust conditions, and *M* as the average effective molecular weight for chamber and exhaust conditions.

- The **isentropic exponent** is the same as *k* or CP/CV for a perfect gas such that PV^k = constant (P=pressure; V=volume). As the gas is not perfect, the values of IS EX and CP/CV do not agree.

- **T* and P*** are the so-called *critical* values of the flow temperature and pressure where the flow velocity is mach one, that is, at the throat. These may be calculated from equations 4 & 6 of the Nozzle Theory Web Page. Units are Kelvin and atmospheres, respectively.

$$T^* = \frac{T_o}{1+\frac{k-1}{2}} \qquad \text{and} \qquad P^* = \frac{P_o}{\left(1+\frac{k-1}{2}\right)^{\frac{k}{k-1}}} \qquad \text{equations 8 \& 9}$$

- **C*** is the Characteristic Exhaust Velocity (cee-star), with units of feet/sec. This parameter may be considered to be a figure of thermochemical merit for a particular propellant, and is given by equation 3 of the Impulse and C* Theory Web Page:

$$c^* = \sqrt{\frac{R'/M\;\;T_o}{k\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}}} \qquad \text{equation 10}$$

- **ISP*** is the vacuum impulse that would be obtained by a sonic nozzle in air-breathing motor work, and thus may be ignored.

- **OPT-EX**, the Optimum Expansion Ratio (Ae/At) is an important parameter in nozzle design. This value defines the ratio of the nozzle exit area to throat area, and as such, sizes the divergence cone exit diameter, where $D_e = D_t \sqrt{A_e/A_t}$. This ratio may be determined from equation 14 of the Nozzle Theory Web Page:

$$\frac{A_e}{A^*} = \frac{1}{\left(\frac{k+1}{2}\right)^{\frac{1}{k-1}} \left(\frac{P_e}{P_o}\right)^{\frac{1}{k}} \sqrt{\left(\frac{k+1}{k-1}\right)\left[1-\left(\frac{P_e}{P_o}\right)^{\frac{k-1}{k}}\right]}} \qquad \text{equation 11}$$

  where k is the value of CP/CV for exhaust conditions.

- The Density Specific Impulse, **D-ISP**, is an interesting parameter. It is defined as the product of specific impulse and the propellant specific gravity, or $I_d = Isp\ \delta_p$ (the specific gravity is numerically equal to the density, in gram/cc). A high value of Density Isp would be important for compact motor designs, where volume is at a premium.

- **A*M** ("A-star M") is the ratio of nozzle throat area to mass flow rate expressed as in^2-sec/lb. I really don't know what this is meant to be used for...!

- **EX-T** is the nozzle exit plane temperature (Kelvin) and may be determined from *equation 5* shown earlier.

Comparison of Performance Equations to GUIPEP

The following table shows an interesting comparison between the results presented by GUIPEP to the same results as calculated by use of the performance equations presented above, which are considered to be "approximate". Nevertheless, the results are in very close agreement.

Parameter		Eqn.	Calculated	GUIPEP
Characteristic velocity	c*	10	2966	2968
Specific Impulse	Isp	7	151.1	151.6
Opt. Expansion ratio	Ae/At	11	10.22	10.22
Critical temperature	T*	8	1629	1625
Critical pressure	P*	9	39.38	39.31
Exit plane temperature	Te	5	1058	1057

Limitations of GUIPEP

To some extent, the accuracy of the results is dependant upon the JANNAF.DAT file which contains reaction species heat of formation data used by the solver. The species list is limited in scope, and for unusual propellant combinations, the actual reaction products may not be present in the list. The result is a failure of the solver, or inaccurate results. A good example is Zinc-Sulphur propellant, for which GUIPEP does not provide any solution. The reason is that the main product of combustion, zinc sulphide, is not present in the list of reaction species.

As mentioned in the introduction, propellant *burn rate* is not assessed by GUIPEP, nor is there any indication provided as to whether a particular propellant concoction will be *self-combusting*. Although it is obvious that this sort of assessment is beyond the scope or intent of GUIPEP, this

fact must be kept in mind when evaluating a propellant. A good example is ammonium nitrate (AN) based propellants. Although GUIPEP typically presents glowing numbers for performance, in reality, the burn rate is usually so slow that the propellant will self-extinguish. Also, the addition of metals such as aluminum is found to boost performance significantly for many propellants, according to GUIPEP results. This is usually not the case in reality, where much of the metal is left unburned unless the propellant reaction temperature is very high and the metal particle size is very fine. Physical limitations also may negate a potentially promising propellant. High solids loading is often predicted to improve performance, but in practice, is usually difficult to achieve due to binder adhesion limitations.

Another limitation, or shortcoming, relates to the prediction of performance of propellants with significant percentage of condensed phase particles in the exhaust (two-phase flow). The value of CP/CV and the isentropic exponent used by the GUIPEP solver for determination of all the performance parameters are calculated for a gas-particle *mixture*, as shown in *Equation 1* of the Two-phase Flow Theory Web Page. However, for flow through the nozzle, a modified isentropic exponent should be used, as given by *Equation 2* in the referenced web page. For propellants with minimal condensed-phase fraction (say, <10%), the overall effect is probably negligible. But for a propellant such as KN-Sucrose, where the condensed-phase fraction is very high (44%), the net effect is more significant. As an example, the chamber value of the isentropic exponent as calculated by *Equation 2* is k=1.04, whereas the value given by *Equation 1* and GUIPEP is k= 1.13. The difference in Ideal Specific Impulse is Isp= 166 sec. versus Isp= 153 sec., respectively.

Last updated August 19, 2001

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Rocket Theory Appendices

Appendix A

Example -- Calculation of the Adiabatic Flame Temperature (AFT) of KN/Sucrose, 65/35 O/F ratio

Consider the combustion of the KN/Sucrose , 65/35 O/F propellant to have the following combustion equation:

$$C_{12}H_{22}O_{11} + 6.288\ KNO_3 \rightarrow 3.796\ CO_2 + 5.205\ CO + 7.794\ H_2O + 3.065\ H_2 + 3.143\ N_2 + 2.998\ K_2CO_3 + 0.274\ KOH$$

The enthalpies of formation for the reactants are obtained from the CRC Handbook of Chemistry and Physics, and for the products, from the JANAF thermochemical tables: (units are kJ/mole)

$C_{12}H_{22}O_{11}$	-2222.1
KNO_3	-494.63
CO_2	-393.52
CO	-110.53
H_2O	-241.83
H_2	0

N_2	0
K_2CO_3	-1150.18
KOH	-424.72

Using the energy balance equation (assuming no changes in K.E. or P.E.):

$$\sum_R n_i\left[h_f + \Delta h\right]_i = \sum_P n_e\left[h_f + \Delta h\right]_e$$

Substituting in the values for h_f , n_i and n_e gives :

$1\ (-2222.10 + 0) + 6.288(-494.63 + 0) = 3.796(-393.52 + \Delta h_{CO2}) + 5.205(-110.53 + \Delta h_{CO}) + 7.794(-241.83 + \Delta h_{H2O}) + 3.065(0 + \Delta h_{H2}) + 3.143\ (0 + \Delta h_{N2}) + 2.998\ (-1150.18 + \Delta h_{K2CO3}) + 0.274\ (-424.72 + \Delta h_{KOH})$

Expanding and gathering terms simplifies the equation to the following form:

$2186.2 = 3.796\Delta h_{CO2} + 5.205\ \Delta h_{CO} + 7.794\ \Delta h_{H2O} + 3.065\ \Delta h_{H2} + 3.143\ \Delta h_{N2} + 2.998\ \Delta h_{K2CO3} + 0.274\ \Delta h_{KOH}$

Solution of the equation is obtained by simply substituting in values for Δh at a certain temperature. This temperature is equal to the AFT when the the right hand side of the equation is equal to the left hand side (=2186.2).

Take a guess that the AFT lies somewhere between 1700 K and 1800 K (easy for me to guess, as I know the answer! But no matter what the guess, the answer will eventually converge).

From the JANAF tables, the values of Δh are: (units are kJ/mole)

T	CO_2	CO	H_2O	H_2	N_2	K_2CO_3	KOH
1700 K	73.480	45.945	57.758	42.835	45.429	280.275	116.505
1800 K	79.431	49.526	62.693	46.169	48.978	301.195	124.815

For the term on the right side of the equation, substituting in the values at T=1700K :

3.796 (73.480) + 5.205 (45.945) + 7.905 (57.758) + 3.065 (42.835) + 3.143 (45.429) + 2.998 (280.275) + 0.274 (116.505) = **2114.5 kJ/mole**

Substituting in the values at T=1800 K:

3.796 (79.431) + 5.205 (49.526) + 7.794 (62.693) + 3.065 (46.169) + 3.143 (48.978) + 2.998 (301.195) + 0.274 (124.815) = **2280.6 kJ/mole**

Clearly, the actual temperature lies in between 1700 and 1800 K. The actual value may be found by using linear interpolation:

$$T_{AFT} = \frac{2186.2 - 2114.5}{2280.6 - 2114.5}(1800 - 1700) + 1700 = \mathbf{1743}\ \mathrm{K}$$

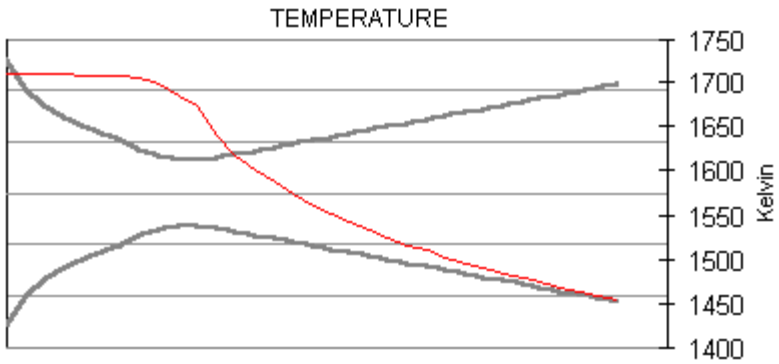
This is in close agreement with the combustion temperature predicted by GUIPEP (1720 K.), that being about 1% lower. The small deviation is a result of the simplified combustion equation assumed in this example. In reality, some trace products such as NH_3 and monatomic K form, consuming energy in the process.

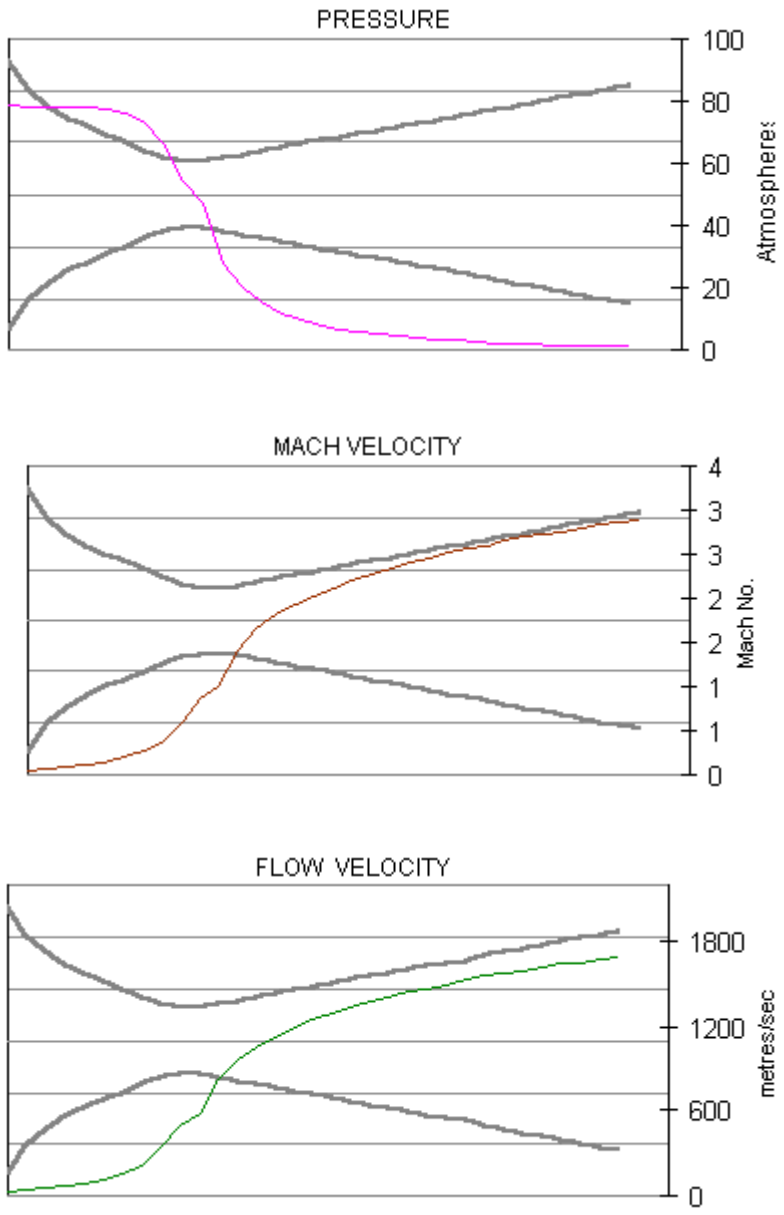
Appendix B

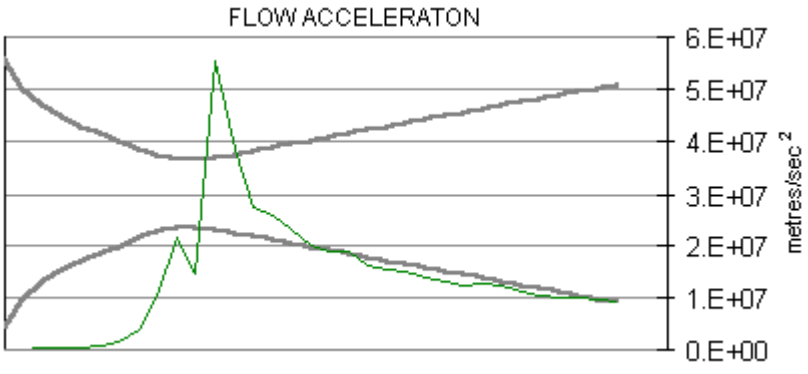
Reserved for future use.

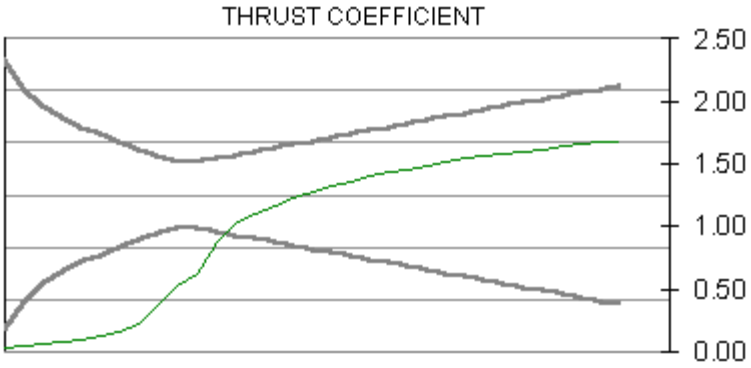
Appendix C

The following are plots of the nozzle flow properties for the Kappa-DX rocket motor:




PRESSURE
100
80
60
40
20
0
Atmospheres
MACH VELOCITY
4
3
3
2
2
1
1
0
Mach No.
FLOW VELOCITY
1800
1200
600
0
metres/sec





Time for flow to travel through nozzle = 430 microseconds.

Appendix D

The derivation of the expression for **mass flow rate through the nozzle** is presented here.
From *Equation 9* of the Nozzle Theory Web Page, the continuity equation for mass flow rate through the nozzle is given by:

$$\dot{m} = \rho^* \, v^* \, A^*$$

where * designates *critical* (throat) conditions. From *Equation 7* of the referenced web page, the critical flow density may be written as:

$$\rho^* = \frac{\rho_o}{\left(1 + \frac{k-1}{2}\right)^{\frac{1}{k-1}}} = \frac{\rho_o}{\left(\frac{k+1}{2}\right)^{\frac{1}{k-1}}}$$

and from *Equations 3 & 4*, the critical (sonic) velocity may be given by:

$$v^* = \sqrt{\frac{2k}{k+1} R T_o}$$

From the ideal gas law, the chamber density may be expressed as:

$$\rho_o = \frac{P_o}{R\,T_o}$$

Substitutionof this equation and those for critical density and velocity into the mass flow rate expression gives:

$$\dot{m} = \frac{P_o}{R\,T_o} \frac{\sqrt{\frac{2k}{k+1} R\,T_o}}{\left(\frac{k+1}{2}\right)^{\frac{1}{k-1}}} A^*$$

which may be rearranged to the form of the expression shown as Equation 4 of the Chamber Pressure Theory Web Page:

$$\dot{m}_n = P_o A^* \sqrt{\frac{k}{R\,T_o}} \left(\frac{2}{k+1}\right)^{\frac{k+1}{2(k-1)}}$$

Appendix E

Example: Calculate the maximum steady-state chamber pressure for the design of the Kappa-DX rocket motor.

Units of measure:
The most prudent (botch-proof) system of units is *mks* (metre : kilogram : second), however, for this example, appropriate English units will used, as well.

Equation 12 of the Chamber Pressure Theory Web Page:

$$P_o = K_n \rho_p\ r\ c^*$$

Burn/throat area	Kn, max. = 378
Propellant density	ρ_p = 1.806 g/cm^3 = 1806 kg/m^3 = 0.00203 slug/in^3
Burn rate	r = 12.65 mm/s = 0.01265 m/s = 0.50 in/s
Propellant c-star	c* = 912 m/s = 2992 ft/s

Therefore,

P_o = 378 (1806) .01265 (926) = 7.9 x 10^6 N/m^2 (7.9 MPa)
or
P_o = 378 (0.00203) 0.50 (2992) = 1148 psi

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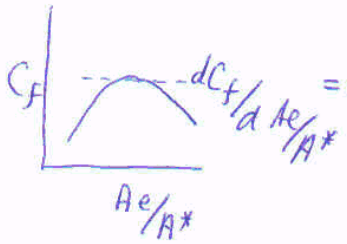
R-NAKKA
DEC-2006

THRUST, $F = C_f P_o A^*$

FOR A GIVEN P_o, F_{max} OCCURS WHEN C_f IS MAX.

C_f IS A FUNCTION OF A_e/A^*



$$C_f = C_1 + \frac{A_e}{A^*}\left(C_2 - \frac{P_a}{P_o}\right)$$

WHERE
$C_1 = \sqrt{\text{term}}$
$C_2 = P_e/P_o$ (FOR ANY FIXED PRESSURE RATIO)

$$= C_1 + \frac{A_e}{A^*} C_2 - \frac{A_e}{A^*}\frac{P_a}{P_o}$$

$$\frac{dC_f}{d\,A_e/A^*} = C_2 - \frac{P_a}{P_o} = 0 \quad \text{at } C_{f\,max}$$

$$\therefore \frac{P_e}{P_o} - \frac{P_a}{P_o} = 0$$

$$\text{or } \frac{P_e - P_a}{P_o} = 0$$

$$\therefore P_e = P_a \text{ at } C_{f\,max} \text{ (or } F_{max})$$

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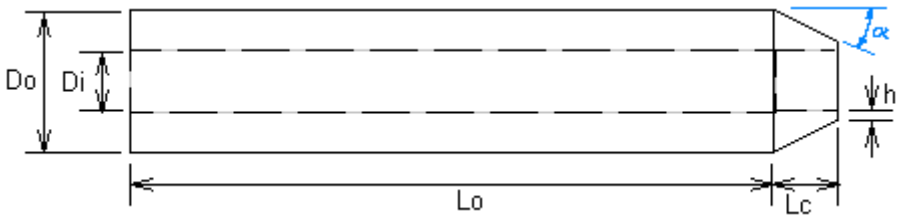
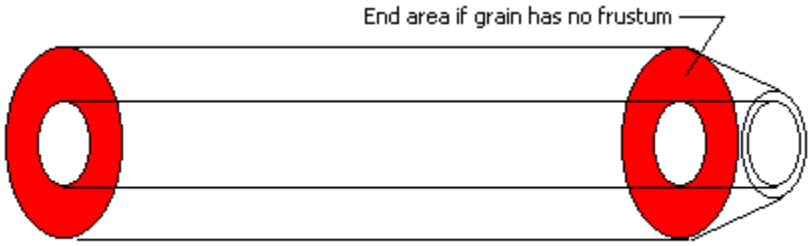
Grain Area Calculations



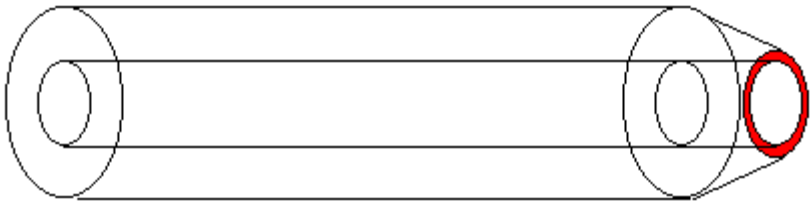
Propellant grain basic dimensions -- Hollow cylindrical grain (may or may not have frustum)

Outer surface area is given by $A_o = \pi D_o L_o$

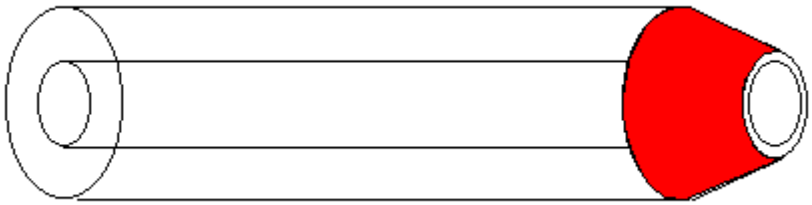
Core surface area is given by $A_c = \pi D_i (L_o + L_c)$



Surface area of *each* end is given by $A_e = \pi (r_o^2 - r_i^2)$



Surface area of the frustum end is given by $A_{fe} = \pi \left[(r_i + h)^2 - r_i^2 \right]$



Surface area of the frustum is given by $A_f = \pi s (r_o + r_p)$

Other terms in the equations are defined as

$r_o = D_o/2 \quad r_i = D_i/2 \quad \text{and} \quad r_p = D_i/2 + h$

$L_c = s \cos \alpha \quad \text{where} \quad s = \frac{r_o - r_p}{\sin \alpha}$

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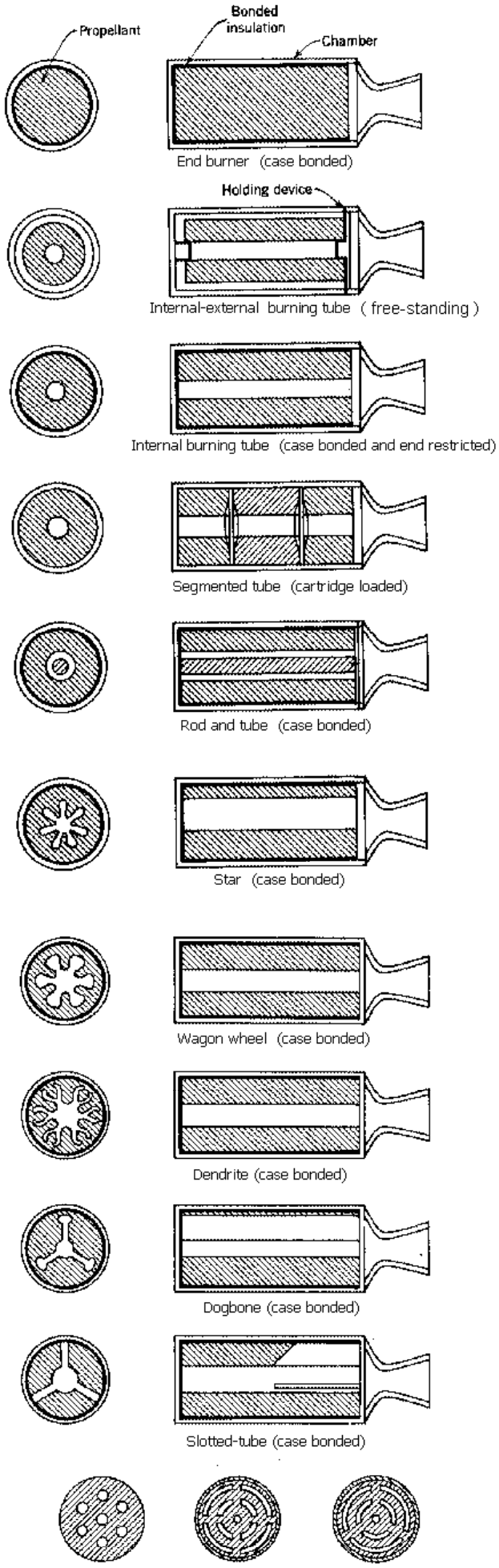

Ref. Hill & Peterson, Mechanics and Thermodynamics of Propulsion

Richard Nakka's *Experimental Rocketry* Web Site

Notes

1. One of the more interesting reactions of this type is the so-called "water gas" reaction:

   $$CO_2 + H_2 <-> CO + H_2O$$

   The equilibrium constant associated with this equation is such that at lower combustion temperatures the reaction proceeds toward the left. This is the reason why there is a significant amount of H_2 produced in the combustion of the KNO3-sucrose propellant (at the 65/35 O/F ratio, the combustion temperature is relatively low, being 1720 K.).

   Water gas, a mixture of H_2 and CO, was used in by-gone days as a household fuel for gas stoves, and was produced by utilities through a process involving passing steam over a bed of hot coke. Since CO (carbon monoxide) is particularly poisonous as well as odourless, deaths were not uncommon as a result of unintended (and sometimes otherwise) exposure.

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Richard Nakka's *Experimental Rocketry* Web Site

Propellant Mass Density Calculation

Mass density of some Rocket Propellant Oxidizers					
		Density			
Solid Oxidizer	**Formula**	g/cm^3	kg/m^3	lb/in^3	slug/in^3
Potassium Nitrate	KNO_3	2.109	2109	0.0762	0.002366
Ammonium Nitrate	NH_4NO_3	1.725	1725	0.0623	0.001935
Potassium Perchlorate	$KClO_4$	2.520	2520	0.0910	0.002827
Ammonium Perchlorate	NH_4ClO_4	1.950	1950	0.0704	0.002188
Sodium Nitrate	$NaNO_3$	2.261	2261	0.0817	0.002537
Sodium Perchlorate	$NaClO_4$	2.018	2018	0.0729	0.002264
Nitronium Perchlorate	NO_2ClO_4	2.198	2198	0.0794	0.002466

Mass density of some Rocket Propellant Fuels					
		Density			
Solid Fuel	**Formula**	g/cm^3	kg/m^3	lb/in^3	slug/in^3
Sucrose	$C_{12}H_{22}O_{11}$	1.5805	1581	0.0571	0.001773
Dextrose (anhydrous)	$C_6H_{12}O_6$	1.562	1562	0.0564	0.001752
Sorbitol (anhydrous)	$C_6H_{14}O_6$	1.489	1489	0.0538	0.001671
Aluminum	Al	2.70	2700	0.0975	0.003029
PVC	-	*1.38	1380	0.0499	0.001548

* Average value. Typical range is from 1.25 to 1.5

References:

1. Merck Index, 7th Ed.
2. CRC Hdbk . of Chemistry and Physics, 54th Ed.
3. Sutton, Rocket Propulsion Elements, 5th Ed.
4. www.matweb.com

Example:
Calculate the ideal density of 65/35 O/F KN-sucrose propellant:

$$\rho_p = \frac{1}{\frac{0.65}{2.109} + \frac{0.35}{1.5805}} = 1.888\ g/cm^3$$

This compares to an actual (measured) value of 1.80 gram/cm3.

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