

## Derivation of Equation (3-19) p.59

$$\dot{w} = \frac{A_x p_1}{R} \sqrt{2gJ} \left\{ \frac{C_p}{T_1} \left[ \left( \frac{p_x}{p_1} \right)^{2/k} - \left( \frac{p_x}{p_1} \right)^{\frac{k+1}{k}} \right] \right\}^{1/2} \quad (3-19)$$

from

$$\dot{w}_x = \frac{A_x v_x}{V_x} \quad (3-2)$$

$$C_p = \frac{kR}{(k-1)J} \quad (3-5)$$

$$\frac{T_x}{T_1} = \left( \frac{p_x}{p_1} \right)^{\frac{k-1}{k}} = \left( \frac{V_1}{V_x} \right)^{k-1} \quad (3-6)$$

$$v_x = \sqrt{\frac{2gkRT_1}{k-1} \left[ 1 - \left( \frac{p_x}{p_1} \right)^{\frac{k-1}{k}} \right]} \quad (3-15)$$

$$\dot{w}_x = \frac{A_x}{V_x} \sqrt{\frac{2gkRT_1}{k-1} \left[ 1 - \left( \frac{p_x}{p_1} \right)^{\frac{k-1}{k}} \right]}$$

$$= \frac{A_x}{V_1 \left( \frac{p_x}{p_1} \right)^{1/k}} \sqrt{2gJ C_p T_1 \left[ 1 - \left( \frac{p_x}{p_1} \right)^{\frac{k-1}{k}} \right]} \quad \text{using (3-6) \& (3-5)}$$

$$= \frac{A_x p_1 \left( \frac{p_x}{p_1} \right)^{1/k}}{R T_1} \sqrt{2gJ C_p T_1 \left[ 1 - \left( \frac{p_x}{p_1} \right)^{\frac{k-1}{k}} \right]}$$

$$= \frac{A_x p_1 \left( \frac{p_x}{p_1} \right)^{1/k}}{R} \sqrt{\frac{2gJ C_p}{T_1} \left[ 1 - \left( \frac{p_x}{p_1} \right)^{\frac{k-1}{k}} \right]}$$

$$\dot{w} = \frac{A_x p_1}{R} \sqrt{\frac{2gJ C_p}{T_1} \left[ \left( \frac{p_x}{p_1} \right)^{2/k} - \left( \frac{p_x}{p_1} \right)^{\frac{k+1}{k}} \right]}$$