Derivation of Equation (3-29) page 66

$$\boxed{F = A_t\, p_1 \sqrt{\frac{2k^2}{k-1}\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}\left[1-\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}\right]} + (p_2 - p_3)\, A_2}$$

Using

$$F = \frac{v_2}{g}\dot{w} + (p_2 - p_3) A_2 \qquad (3\text{-}28)$$

$$\dot{w} = \frac{A_t v_t}{V_t} \qquad (\text{continuity})$$

$$v_2 = \sqrt{\frac{2gk}{k-1} R T_1 \left[1-\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}\right]} \qquad (3-15)$$

$$V_t = V_1 \left(\frac{k+1}{2}\right)^{1/k-1} \qquad (3\text{-}21)$$

$$v_t = \sqrt{\frac{2gk}{k+1} R T_1} \qquad (3\text{-}23)$$

$$F = \frac{A_t v_t}{V_t\, g}\sqrt{\frac{2gk}{k-1} R T_1\left[1-\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}\right]} + (p_2 - p_3) A_2$$

$$= \frac{A_t}{V_t\, g}\sqrt{\frac{4g^2k^2R^2T_1^2}{(k-1)(k+1)}\left[1-\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}\right]} + (p_2 - p_3) A_2$$

since $V_t = \frac{RT_1}{p_1}$

$$= \frac{A_t}{V_1\left(\frac{k+1}{2}\right)^{1/k-1}}\sqrt{\frac{4k^2R^2T_1^2}{(k-1)(k+1)}\left[1-\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}\right]} + (p_2 - p_3) A_2$$

since $V_1 = \frac{R \cdot T_1}{p_1}$

$$= \frac{A_t\, p_1}{\left(\frac{k+1}{2}\right)^{1/k-1}}\sqrt{\frac{4k^2}{(k-1)(k+1)}\left[1-\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}\right]} + (p_2 - p_3) A_2$$

$$= A_t\, p_1 \sqrt{\frac{4k^2}{\left(\frac{k+1}{2}\right)^{2/k-1}(k-1)(k+1)}\left[1-\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}\right]} + (p_2 - p_3) A_2$$

since $\left(\frac{k+1}{2}\right)^{2/k-1}(k+1) = 2\left(\frac{k+1}{2}\right)^{\frac{k+1}{k-1}}$ [see derivation of (3-24)]

$$= A_t p_1 \sqrt{\frac{2k^2}{(k-1)\left(\frac{k+1}{2}\right)^{\frac{k+1}{k-1}}}\left[1-\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}\right]} + (p_2 - p_3)A_2$$

$$\Rightarrow \boxed{F = A_t p_1 \sqrt{\frac{2k^2}{k-1}\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}\left[1-\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}\right]} + (p_2 - p_3)A_2}$$