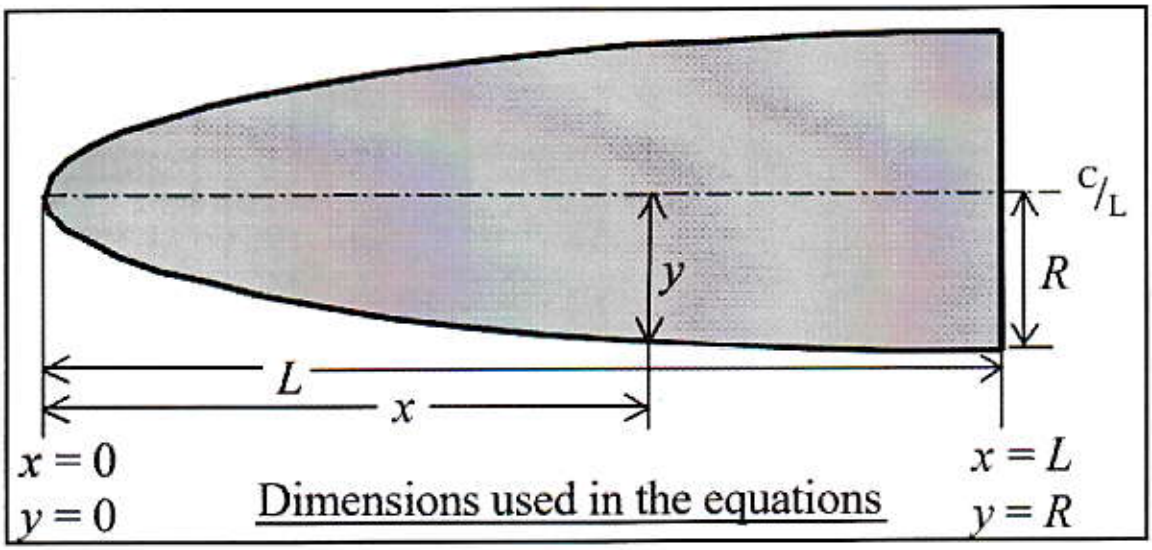


Dimensions used in the equations

For $0 \le n \le 1$, $\quad y = R\left(\frac{x}{L}\right)^n$

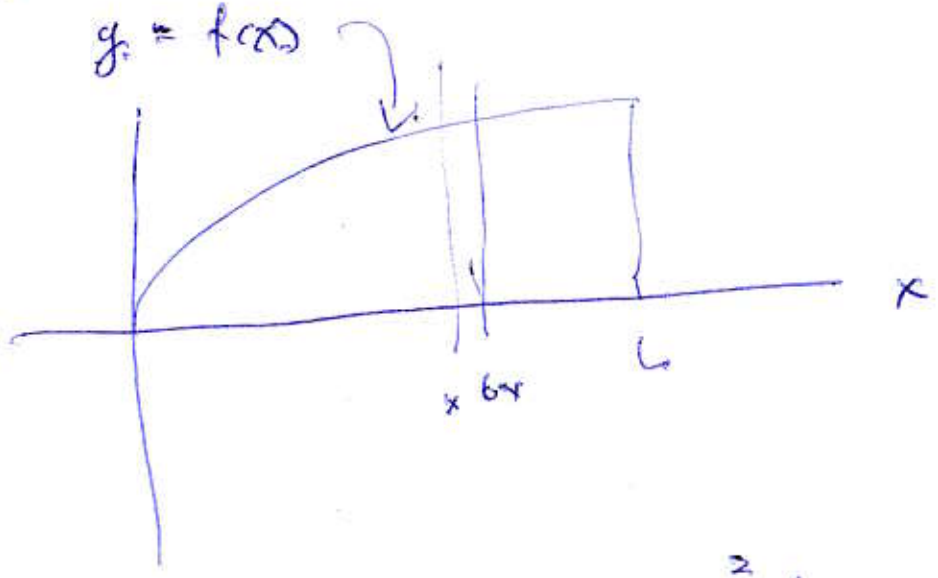
Where: n = 1 for a **CONE**

n = .75 for a ¾ **POWER**

n = .5 for a ½ **POWER (PARABOLA)**

n = 0 for a **CYLINDER**

Volumes



$f(0) = 0$

$f(L) = R$

$$\delta A = \pi y^2 dx = \pi f(x)\, dx$$

$$\delta V = \pi y^2 \delta x = \pi f^2(x)\, \delta x$$

$$V = \int_0^L \pi f^2(x)\, dx = \int_0^L \pi R^2 \left(\frac{x}{L}\right)^{2n} \delta x$$

where $f(x) = R\left(\frac{x}{L}\right)^n$

(2)

$$V = \pi R^2 \int_0^L \frac{x^{2n}}{L^{2n}}\, dx$$

$$= \frac{\pi R^2}{L^{2n}} \int_0^L x^{2n}\, dx$$

$$= \frac{\pi R^2}{L^{2n}} \left. \frac{x^{2n+1}}{2n+1} \right|_0^L$$

$$= \frac{\pi R^2}{L^{2n}} \frac{L^{2n+1}}{2n+1} = \frac{\pi R^2 L}{2n+1} \quad (1)$$

Centroid

weighted factor

$$V x_{centroid} = \int_0^L \pi y^2 x\, dx$$

$$= \int_0^L \pi R^2 \left(\frac{x}{L}\right)^{2n} x\, dx$$

$$x_{cent} = \frac{1}{V} \int_0^L \frac{\pi R^2 x^{2n}}{L^{2n}} \times dx$$

$$= \frac{\pi R^2 L}{2n+1} \cdot \frac{2n+1}{\pi R^2 L} \cdot \frac{\pi R^2}{L^{2n}} \int_0^L x^{2n+1} dx$$

$$= \frac{2n+1}{L^{2n+1}} \left[\frac{x^{2n+2}}{2n+2} \right]_0^L$$

$$= \frac{2n+1}{L^{2n+1}} \cdot \frac{L^{2n+2}}{2n+2}$$

$$= \left(\frac{2n+1}{2n+2} \right) L \quad \cdots (2)$$

Ex

Cylinder $n = 0$

$$x_{cent} = \frac{L}{2}$$